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ORIGINAL ARTICLE

Numerical Study of Hydraulic Jump on Rough Beds Stilling Basins

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ABSTRACT: The hydraulic jump phenomenon is one of the most common phenomena in open channels. A hydraulic jump is a rapid transition from a supercritical flow into a subcritical flow by dissipating a lot of energy. Stilling basins bottom roughness is an important factor to control, reducing jump length and increase energy losses by hydraulic jump. In present study, 2-dimensional numerical simulation of hydraulic jump on rough beds including triangular and rectangular sheets by using standard k-ε model was evaluated. The free surface was determined using the VOF method. Total of 16 simulations were simulated for a range of the Froude number from 3 to 7. The results showed that the sequent depth and length of hydraulic jump are reduced while the energy loss is increased, comparing with classic jump. Also there was a good agreement between modelled results and experimental data. The investigation resulted in some empirical relations to define the sequent depth and bed shear stress for different flow conditions on used rough beds.

Keywords: Hydraulic jump, Rough beds, Standard k-ε model, VOF method

INTRODUCTION

Hydraulic jump has been used for dissipation of kinetic energy downstream of hydraulic structures such as spillways, chutes and gates. A jump formed in horizontal, wide rectangular channels with smooth bed is called classic jump and has been widely studied by Peterka (1958), Rajaratnam (1967), McCorquodale (1986) and Hager (1992). Hydraulic jump can be controlled by roughness shapes in stilling basins bottom. Recently, some investigations have been carried out on hydraulic jump on rough beds. Many different roughness shapes in basin bottom have been studied experimentally by (Izadjoo and Shafai-bejestan, 2005), Ead (2007) and (Elsebaie and Shabayek, 2010).

Many researchers have been evaluated numerical study of hydraulic jump on rough beds stilling basins. As the flow is two-phase turbulent flow in a hydraulic jump, so simulation of the hydraulic jump by using turbulence models and VOF (volume-of-fluid) method can be led to accurate results. Gharangik and Chaudhry (1991) investigated numerical model of the hydraulic jump. They applied the Boussinesq equations to simulate both the sub and supercritical flows and a hydraulic jump in the rectangle channel having a small bed slope. The numerical study of hydraulic jump on a smooth bed has been done by Zhao and Misra (2004). The governing equations are the continuity and momentum equations for incompressible flow and based on the two-dimensional k-ε turbulence models. The results using the VOF and the scale turbulence model involving water surface location, horizontal velocity, turbulent kinetic energy (TKE) profiles and the energy dissipation were presented. Sarker and Rhodes (2000) studied hydraulic jump on smooth bed by physical and numerical methods. The RNG k-ε turbulence model was used, in combination with the volume of fluid (VOF) method for free surface modelling. Good agreement was obtained between the 2-dimensional CFD solution and the physical measurements. Gonzalez and Bombardelli (2005) simulated a hydraulic jump on smooth bed by using k-ε turbulence and Large Eddy Simulation models. Results are compared with observations of mean flow and turbulence in hydraulic jumps by Liu et al. (2004).

The objective of this research is to investigate the numerical model of hydraulic jump on rough beds including triangular and rectangular sheets using standard k-ε turbulence model. This study was done with a CFD program which uses the finite-volume method to solve 2D Reynolds-averaged Navier Stokes (RANS) equations. In this program, the free surface location is computed using VOF (volume-of-fluid) method.

MATERIALS AND METHODS

Governing equations

The governing equations are the unsteady incompressible two-dimensional continuity and Reynolds-averaged Navier-Stokes equations for the liquid and the air (Liu et al., 2002).

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_i}(\rho U_i)=0 \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho U_j)+\frac{\partial}{\partial x_i}(\rho U_i U_j)=-\frac{\partial P}{\partial x_j}+\frac{\partial}{\partial x_i}(\mu+\mu_t)\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)+\rho g_j \qquad (2)$$

$$\rho=\alpha_A\rho_A+\alpha_w\rho_w \qquad (3)$$

where U_i are the velocity components, α_A and α_w are the volume fraction of air and water respectively; ρ_A,

ρ_w and ρ are the density of air, water and mixture respectively; p is the pressure and g is the gravity acceleration. The turbulent viscosity, μ_t is computed by combining k and ε as follows:

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \quad (4)$$

$$\mu = \alpha_A \mu_A + \alpha_w \mu_w \quad (5)$$

The parameters of μ, μ_A, μ_w and μ_t, are the viscosity of mixture, air, water and the eddy viscosity respectively.

Experimental procedure

In this study, the reported measurement by Elsebaie and Shabayek (2010) were used. Hydraulic jumps were produced in a rectangular flume 0.295 m wide, 0.32 m deep and 9 m long.

Rough beds wooden triangular and rectangular sheets were installed on the flume bed in such a way that the crests of rough beds were at the same level as the upstream bed on which the supercritical stream was produced by sluice gate. Two types of rough beds (I, II) had a wavelength s of 0.065 m in the flow direction and on amplitude t of 0.018 m and the triangular section had slide slopes of 45 degrees. A tailgate was used to control the tailwater depth in the flume. The primary details of these tests are shown in table 1. Supercritical depth $y_{1,}$ tailwater depth y_2, and the length of the jump L_j, in the experiments of series A, B, C and D are recorded and showed in table 1.

Numerical model

In present research hydraulic jump on rough beds including triangular and rectangular sheets are numerically studied for different Froude numbers using the standard k-ε model and two-phase flow theory.

Volume of fluid (VOF)

The water surface location was determined with the volume of fluid (VOF) method. The VOF method uses a function F(x, y, t) to assign the free surface. The function F is obtained from the following equation:

$$\frac{\partial F}{\partial t} + U_j \frac{\partial F}{\partial x_j} = 0 \quad (6)$$

A unit value of F corresponds to a cell full of fluid, while a zero value indicates that the cell contains no fluid. Cells with F values between zero and one contain a free surface. The function F can be solved with different methods. In this study Geometric Reconstruction method was used. The geometric reconstruction scheme represents the interface between fluids using a piecewise-linear approach. This scheme is the most accurate and is applicable for general unstructured meshes. In Figure 1 the interface between two fluids are represented for actual and the geometric reconstruction scheme.

Standard k-ε model

In the present study in order to model the hydraulic jump, standard k-ε turbulence model were used.

Standard *k-ε* model is given by the following equations:

$$\rho U_i \frac{\partial k}{\partial x_i} = \mu_t\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right)\frac{\partial U_j}{\partial x_i} + \frac{\partial}{\partial x_i}\left\{(\mu_t/\sigma_k)\frac{\partial k}{\partial x_i}\right\} - \rho\varepsilon \quad (7)$$

$$\rho U_i \frac{\partial \varepsilon}{\partial x_i} = C_{1\varepsilon}\left(\frac{\varepsilon}{k}\right)\mu_t\left(\frac{\partial U_j}{\partial x_i} + \frac{\partial U_i}{\partial x_j}\right)\frac{\partial U_j}{\partial x_i} + \frac{\partial}{\partial x_i}\left\{(\mu_t/\sigma_\varepsilon)\frac{\partial \varepsilon}{\partial x_i}\right\} - C_{2\varepsilon}\rho\left(\frac{\varepsilon^2}{k}\right) \quad (8)$$

where k and ε are the turbulent kinetic energy and dissipation rate. σ_k=1 and σ_ε=1.3 are the turbulent Prandtl numbers for k and ε respectively. C_1= 1.44 and C_2= 1.92 are empirical constants.

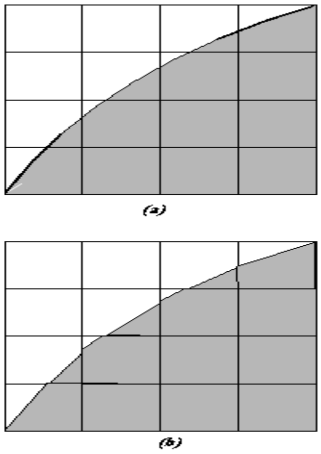

Figure 1. Interface calculations; (a) actual interface shape; (b) the geometric reconstruction scheme

Model geometry and boundary conditions

To analyze the hydraulic jump on rough beds the model geometry were generated and the unstructured grids are created. The grids size in the model geometry was 5 mm. The view sketch of the initial regions and boundary conditions in the jump on rough beds were shown in Figure 2.

The basic procedure is to define primary phase water and a secondary phase air as shown in Figure 2. According Figure 2, the inlets were defined as the water flow (AB) and air flow (BC) into the domain. The boundary conditions also were defined as stream wise velocity or hydrostatic pressure inlets (AB), hydrostatic pressure inlet (BC) and outlet (DE) equal zero and wall boundary condition (channel bed and gates). The values of the velocity and the turbulence parameters, i.e., the turbulent intensity and hydraulic diameter were obtained from experimental data. To calculate the effect of the wall on the flow, empirical wall functions known as standard equilibrium were used. The time-dependent solution procedure was given an initial condition in which all of the cells upstream of the gate and up to velocity inlets level were patched as water filled; F =1.

Table 1. Primary detail of experiment (Elsebaie and Shabayek, 2010)

Exp.	sheet*	t (mm)	S (mm)	q (m^2/s)	U_1 (m/s)	y_1 (mm)	Fr_1	Re	y_2 (m)	L_j (m)
A_1	I	18	65	0.062	2.48	25	5.0	61903	0.092	0.230
A_2	I	18	65	0.071	2.84	25	5.7	71122	0.105	0.350
A_3	I	18	65	0.079	3.17	25	6.4	79275	0.121	0.400
A_4	I	18	65	0.0865	3.46	25	7.0	86488	0.141	0.430
B_1	I	18	65	0.105	2.10	50	3.0	105054	0.110	0.200
B_2	I	18	65	0.116	2.32	50	3.3	116141	0.115	0.300
B_3	I	18	65	0.126	2.53	50	3.6	126259	0.120	0.330
B_4	I	18	65	0.135	2.71	50	3.9	135624	0.133	0.350
C_1	II	18	65	0.062	2.48	25	5.0	61903	0.100	0.420
C_2	II	18	65	0.071	2.84	25	5.7	71122	0.120	0.570
C_3	II	18	65	0.079	3.17	25	6.4	79275	0.138	0.780
C_4	II	18	65	0.0865	3.46	25	7.0	86488	0.150	1.000
D_1	II	18	65	0.105	2.10	50	3.0	105054	0.137	0.584
D_2	II	18	65	0.116	2.32	50	3.3	116141	0.149	0.782
D_3	II	18	65	0.126	2.53	50	3.6	126259	0.157	0.913
D_4	II	18	65	0.135	2.71	50	3.9	135624	0.165	1.070

** Sheet I: Triangular rough bed;*
** Sheet II: Rectangular rough bed*

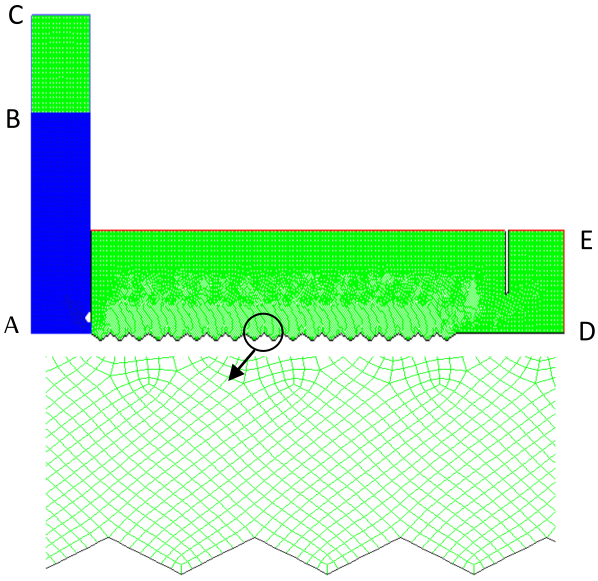

Figure 2. Boundary conditions and initial air and water flow regions

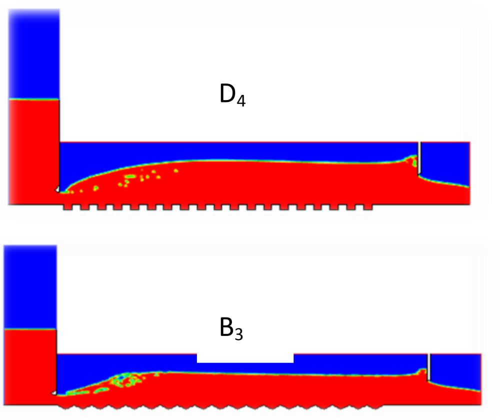

Figure 3. Simulation of water surface profiles using the VOF method for experiments D_4 and B_3

RESULTS AND DISCUSSION

Standard k-ε turbulent model was used to simulate the hydraulic jump on rough beds and the characteristics of jump such as water surface profile, length of the jump, sequent depth and bed shear stress could be evaluated.

Water surface profile

The free surfaces obtained by the volume of fluid (VOF) approach have been shown in Figures 3 and 4. Simulated water surface profiles are agreed well with measured values. The mean relative error of water surface profiles between simulation and experimental values is about 0-4.4 %.

Sequent depth ratio

The relation between initial Froude number Fr_1 and depth ratio y_2/y_1 for roughness shapes in simulation model is plotted, as shown in Figure 5. Belanger equation for classic jump is also shown in Figure 5. Elsebaie and Shabayek (2010) were found that the depth ratio y_2/y_1 is approximately equal to 88% of the supercritical Froude number Fr_1. It can be seen from Figure 5 that the relative roughness have small influence on the sequent depth ratio. Figure 5 shows that a good agreement was found between numerical and experimental results.

Hydraulic jump length

According graphs of water surface profiles (Fig.4), the length of jump L_j can be obtained in the experiments. The mean relative error of the length of jump between simulation and experimental values is about 0-6.7%. Figure 6 shows the values of L_j/y_2^* versus Froude number (Fr_1) for two types of triangular and rectangular beds. The average value of L_j/y_2^* is nearly 2.5.

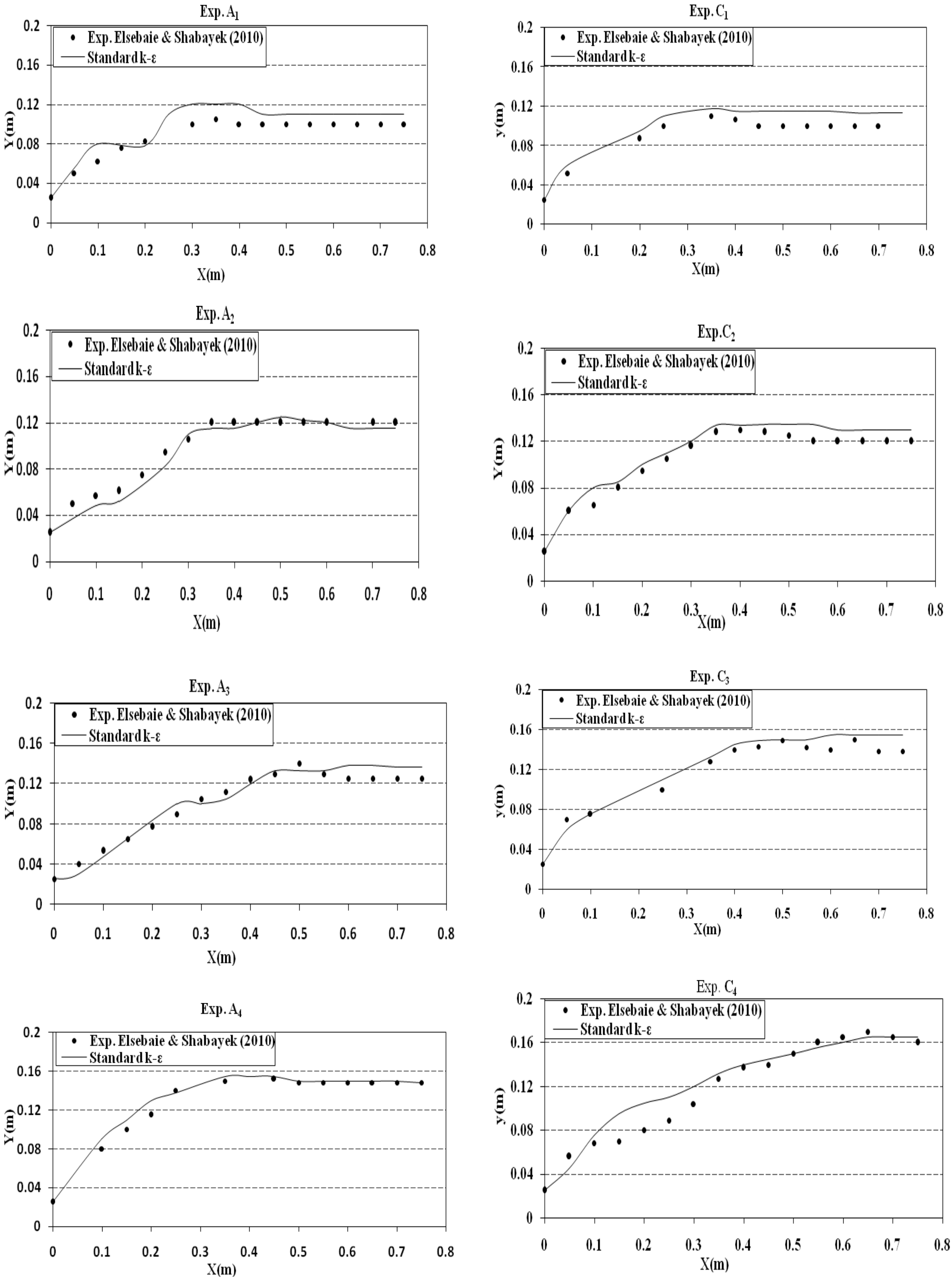

Figure 4. Comparison of free surface profiles between numerical turbulent models and experimental data

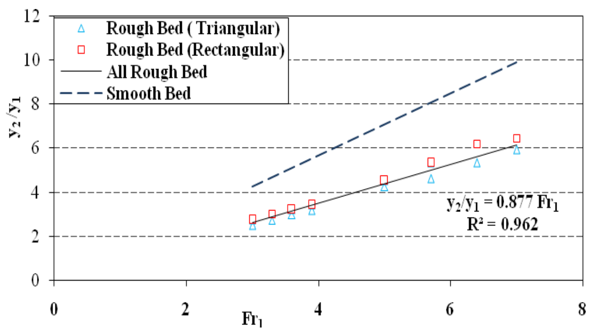

Figure 5. Relation between the depth ratio y_2/y_1 and Fr_1

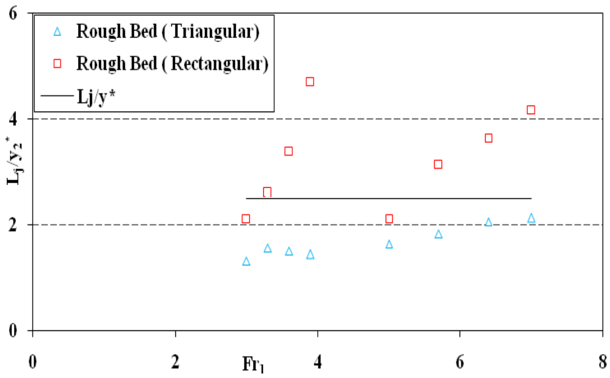

Figure 6. Relation between the ratio of L_j / y_2^* and Fr_1

Bed shear stress

The bed shear stress of jump is obtained using the integral momentum equation. The depth– averaged momentum equation in the longitudinal direction can be derived by integrating Reynolds equation over the depth of flow. The resulting equation can be written as follows:

$$\frac{\partial}{\partial x}\int_o^y \rho u^2 dz + \frac{\partial}{\partial x}\int_0^y p dz - \frac{\partial}{\partial x}\int_0^y \sigma_x dz = -\tau_b \qquad (9)$$

Where ρ= water density; P= hydrostatic pressures σ_x= Reynolds normal stress; and τ_b= bed shear stress. By using the momentum equation at the sections just before and after the jump, the integrated shear force can be written as follows:

$$F_\tau = \int_{x_1}^{x_2} \tau_b dx = (P_1 - P_2) + M_1 - M_2 \qquad (10)$$

Where P_1, P_2, M_1, M_2, and S_1, S_2 are integrated pressures, momentum, and normal shear force per unit width at the sections just before and after the jump (Khan and Steffler, 1996).

The shear force coefficient introduced by Wu and Rajaratnam (1995) was defined as:

$$\varepsilon = \frac{F_\tau}{\gamma y_1^2 / 2} \qquad (11)$$

The shear force coefficient ε was obtained by Equations (10), (11) and using standard k-ε model. In Figure 7 the variation of shear force coefficient ε with Froude number is shown. According to the figure the results were obtained using experimental and turbulent model give good estimates of the shear force coefficient ε. The equation for the shear stress coefficient may be written as:

$$\varepsilon = 1.593 Fr_1^2 - 5.166 Fr_1 + 1.916 \qquad (12)$$

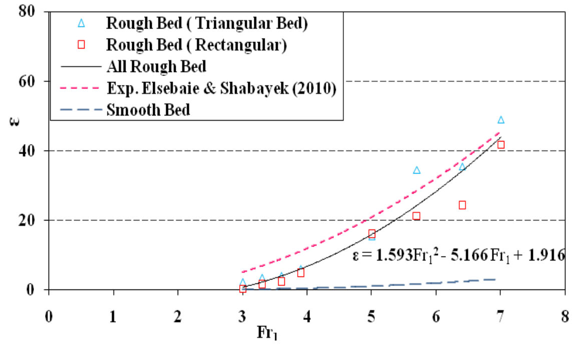

Figure 7. Comparison of shear force coefficient estimated from the turbulent model and experimental data

CONCLUSIONS

The numerical scheme provided by standard k-ε turbulence model was used to predict the 2-D water surface location, sequent depth ratio, length jump and the bed shear stress for jump on rough beds. The computed values were then compared to the experimental values obtained by Elsebaie and Shabayek (2010).

Overall, the computational results showed a close agreement with the various selected experimental results. The water surface profiles can be predicted with an accuracy range of 0-4.4%. The mean relative error of the length of jump between simulation and experimental values is about 0-6.7%. Relation between the ratio of y_2/y_1 and Fr_1 shows that a good agreement was found between numerical and experimental results. In addition using experimental and turbulent model give good estimates of the shear force coefficient ε.

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