

Regional Flood Frequency Analysis Based on L-Moment Approach (Case Study West Azarbayjan Basins)

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ABSTRACT: Flood estimation with certain frequency is one of the fundamental factors for design of Hydraulic structures, Flood plain, River coastal stabling, Basin management, etc. Accurate estimation of flood frequency discharge increases safety of the structures. L-moment approach was used for flood frequency analysis in west Azarbayjan province basins. For identifying homogeneous regions, the Ward hierarchical cluster method was used. Site data were used for independent testing of the cluster of the station for homogeneity. The west Azarbayjan province divided to four regions. In these regions parameters of the regional frequency distribution were evaluated by L-moment ratios. The L-moment diagram, goodness of fit test, and plotting position methods were used for the selection of appropriate distributions. In west Azarbayjan, Generalized Pareto distribution for region A, Generalized extreme value, Pearson type III and Lognormal distributions for region B, Pearson type III, Lognormal, Generalized extreme value distributions for region C and Lognormal and Generalized extreme value distribution were appropriate for region D. The relative Root Mean Square Error (rRMSE) between observed and estimated data in all stations is small. Regional model evaluated for determination of flood discharge magnitude in different frequency by liner and multiple regression method.

Keywords: L-Moment, Regional Frequency, Azerbaijan, Homogeneity

ORIGINAL ARTICLE

INTRODUCTION

An important practical application of hydrology is the estimation of extreme events, especially because the planning and design of water resource projects and flood-plain management depend on the frequency and magnetite of peak discharges. Regional flood frequency analysis is usually applied when no local data are available at a site of interest or the data are insufficient for a reliable estimation of flood quantiles for the required return period. Regional flood frequency analysis has three major components, namely, delineation of homogeneous region, determination of appropriate probability density function (or frequency curves) of the observed data, and the development of a regional flood frequency model (i.e., a relationship between flows of different return periods, basin characteristics, and climatic data). The study includes identification of homogeneous regions based on cluster analysis of site characteristics, identification of suitable regional frequency distribution and development of a regional flood frequency models in west Azarbayjan province basins (Iran).

L-moments

Recently, Hosking (1990) has defined L-moments. Which are analogous to conventional moments, and can be expressed in terms of linear combinations of order statistics? Basically, L-moments are linear functions of probability-weighted moments (PWMs). Similar to conventional moments, the purpose of L-moments and probability-weighted moments is to summaries theoretical

distribution and observed samples. Greenwood et al. (1979) summarizes the theory of PWM and defined them as

$$\beta_r = E\{X[F_X(x)]^r\} \quad (1)$$

Where β_r is the r th order PWM and $F_X(x)$ is the cumulative distribution function of X . Unbiased sample estimators (b_i) of the first four PWMs are given as (Hosking and Wallis, 1997)

$$\beta_0 = m = \frac{1}{n} \sum_{j=1}^n X_j \quad (2)$$

$$\beta_1 = \sum_{j=1}^{n-1} \left[\frac{n-j}{n(n-1)} \right] X_{(j)} \quad (3)$$

$$\beta_2 = \sum_{j=1}^{n-2} \left[\frac{(n-j)(n-j-2)}{n(n-1)(n-2)} \right] X_{(j)} \quad (4)$$

$$\beta_3 = \sum_{j=1}^{n-3} \left[\frac{(n-j)(n-j-1)(n-j-2)}{n(n-1)(n-2)(n-3)} \right] X_{(j)} \quad (5)$$

Where $X_{(j)}$ represents the ranked AMS with $X_{(1)}$ being the highest value and $x_{(n)}$ the lowest value, respectively. The first four L-moment are given as follow:

$$\lambda_1 = \beta_0 \quad (6)$$

$$\lambda_2 = 2\beta_1 - \beta_0 \quad (7)$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0 \quad (8)$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 \quad (9)$$

Unbiased sample estimators of the first four L-moments are obtained by substituting the PWM sample estimators from Equation 2 to 5 into Equation 6 to 9, respectively. The first L-moment λ_0 is equal to the mean value of X. Finally, the L-moment ratios are calculated as:

$$L - C_v = \tau_2 = \frac{\lambda_2}{\lambda_1} \quad (10)$$

$$L - \gamma = \tau_3 = \frac{\lambda_3}{\lambda_2} \quad (11)$$

$$L - k = \tau_4 = \frac{\lambda_4}{\lambda_2} \quad (12)$$

Sample estimates of L-moment ratios are obtained by substituting the L-moments in Eqs. (10 to 12) with sample L-moments.

Index flood

The T-year event X_T is defined as the event exceeded on average once every T years (Stedinger et al., 1993) and is given as

$$P[X > x_T] = \frac{1}{T} \quad (13)$$

When the annual maximum floods are distributed according to a specified frequency distribution with CDF F, the T-year event can be calculated as:

$$X_T = F^{-1}(1 - 1/T) \quad (14)$$

Regional frequency analysis methods, such as the index flood method, include information from nearby stations exhibiting similar statistical behavior as at the site under consideration in order to obtain more reliable estimates. Regional methods can also be used to obtain estimates at ungauged sites, which is important in region such as west Azarbayjan province, where the flow gauging network density is relatively low.

Consider a homogeneous region with N sites, each site i having sample size n_i and observed AMS X_{ij} , $j=1, \dots, n_i$. The AMS from a homogeneous region are identically distributed except for a site-specific scaling factor, viz., the index flood. At each site the AMS is normalized using the index flood as

$$q_{ij} = X_{ij} / \mu_i \quad (15)$$

Where μ_i is the mean annual flood (MAF) at site i, which is often used as the index flood. The sample L-

moment ratios are estimated at each site and the regional record length weighted average L-moment ratios are calculated as

$$\hat{\lambda}_r^R = \frac{\sum_{i=1}^N n_i \hat{\lambda}_r^{(i)}}{\sum_{i=1}^N n_i} \quad (16)$$

Where $\hat{\lambda}_r^{(i)}$ is the rth order sample L-moment ratio at site i, and $\hat{\lambda}_r^R$ is the rth order regional average sample L-moment ratio.

The parameters of a regional frequency distribution can be estimated using the method of L-moment ratios, as shown, for example, by Stedinger et al. (1993) and Hosking and Wallis (1997). Finally, the T-year event at site i can be estimated as:

$$\hat{Q}_{T,i} = \hat{\mu}_i \hat{q}_T \quad (17)$$

Where $\hat{\mu}_i$ is the MAF at site i, and \hat{q}_T is the regional growth curve. The regional growth curve is the (1-1/T)-quantile of the regional distribution of the normalized AMS as defined through Eq. (17).

IDENTIFICATION OF HOMOGENEOUS REGIONS

From hydrometric stations in west Azarbayjan province 62 hydrometric sites, which have sufficient length record and are important for frequency analysis, were selected. For identification of homogeneous regions Hosking and Wallis recommended using Ward's method, which is a hierarchical clustering method based on minimizing the Euclidean distance in site characteristics space within each cluster. The site characteristics selected in this study for each station included: latitude (LAT) and longitude (LON) of the flow gauging weir, mean annual flood (MAF), station area (AREA), altitude (ALT), design storm intensity (ID) and runoff coefficient (C). Using this method west Azarbayjan province divided to four regions (A, B, C and D). Table 1 shows the site characteristic for region A stations. Figure 1 shows the homogenous regions in west Azarbayjan province.

After identification of homogeneous regions, using Hosking's method discordancy measure (D_i) of sites was determinate in each region. Table 2 shows the L-moment ratios and discordancy measure for region B stations.

Table 1. Site characteristics for region A stations.

Station	LAT	LON	ALT (m)	MAF	AREA (km ²)	C	ID(mm/hr)
Gurool olya	43.01	48.27	2158	159.4	1278	0.29	1.9
Badlan	42.43	43.30	2467	39.2	764.8	0.13	2.3
Baroon	43.23	45.31	2171	108.1	991.3	0.1	1.78
Badavi	43.33	43.22	2219	33.9	265.2	0.07	3.76
Makoo	43.54	45.55	1755	13.1	1572	0.13	2.25
Gale joog	43.29	45.27	2159	105.7	1031	0.16	2.4
Gara kurpi	43.31	45.09	632	61.69	471	0.13	2.51
Bashkand	43.33	44.20	2017	24.43	380.4	0.14	2.88

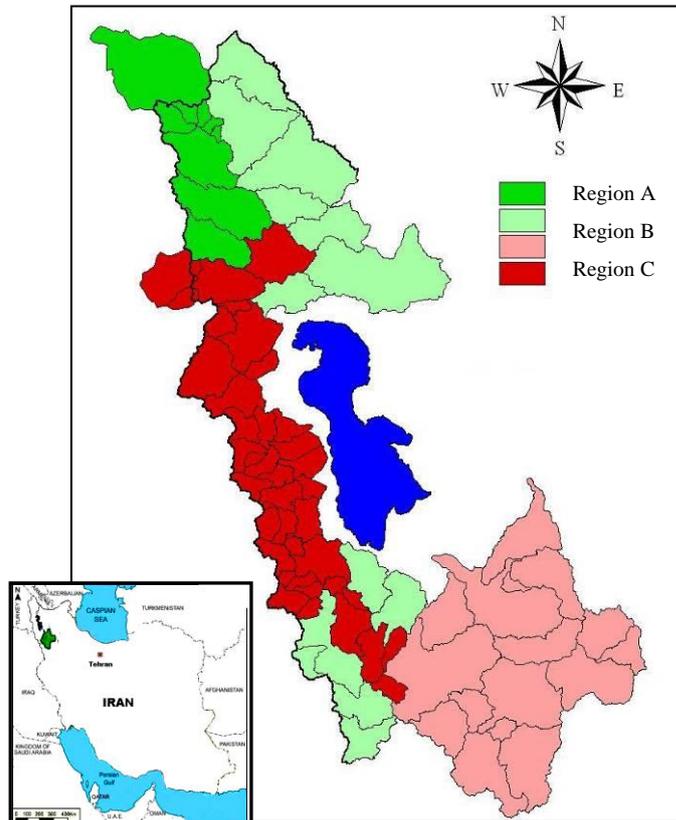


Figure.1 The homogeneous regions in West Azarbaijan Province

Tabl .2 L-moment ratios and discordancy measure for region B stations

Station name	Record length (year)	MAF λ_0 (m3/s)	L-CV	L-SKEW	L-KURT	Di
Poldasht	37	46.54	0.2691	0.1631	0.1531	0.3
Moosa goli	37	49.50	0.4335	0.2473	0.0913	1.82
Barisoo	37	100.48	0.3731	0.2555	0.1939	0.59
Yalgooz aghaj	37	31.22	0.2875	0.1412	0.0774	0.58
Bayazid abad	37	9.35	0.3348	0.4558	0.3352	2.41
Kergal	37	373.72	0.2875	0.1765	0.0633	1.26
Gard yagoob	37	100.15	0.3091	0.1626	0.1488	0.04
Polbahramloo	37	112.56	0.2335	0.0867	0.1802	0.75
Mozafar abad	37	43.73	0.2690	0.0465	0.1765	1.33

Heterogeneity test

Hosking and Wallis (1997) proposed a statistical test based on L-moment ratios for testing the heterogeneity of the proposed regions. The test compares the between-site variation in sample L-CV with the expected variation for a homogeneous region. The method fits a four parameters kappa distribution to the regional average L-moment ratios. The estimated kappa distribution is used to generate 500 homogeneous regions with population parameters equal to the regional average sample L-moment ratios. The properties of the simulated homogeneous region are compared to the sample L-moment ratios as

$$H = (V_1 - \mu_{V_1}) / \sigma_V \quad (18)$$

Where μ_V is the mean of simulated V values, and σ_V is the standard deviation of simulated V values.

For the sample and simulated regions, respectively, V is calculated as

$$V = \left\{ \frac{\sum_{i=1}^N n_i (t^{(i)} - t^R)^2}{\sum_{i=1}^N n_i} \right\}^{\frac{1}{2}} \quad (19)$$

Where N is the number of sites, n_i is the record length at site i, $t_{(i)}$ is the sample L-CV at site I, and t_R is the regional average sample L-Cv.

If $H < 1$, the region can be regarded as 'acceptable homogeneous', $1 \leq H < 2$ is 'possible homogeneous', and $H \geq 2$ is 'definitely heterogeneous' (Hosking and Wallis, 1997). Table 3 shows the heterogeneity measure for identified regions in west Azarbaijan province.

Table.3 Heterogeneity measure for identified regions in west Azarbaijan province.

SI No	Heterogeneity measure	Valuse			
		Region A	Region B	Region C	Region D
1	Heterogeneity measure H₁				
	a) Observed standard deviation of group L-Cv	0.0498	0.0578	0.0396	0.0701
	b) Simulated mean of standard deviation of group L-Cv	0.0377	0.0370	0.0416	0.0487
	c) Simulated standard deviation of standard deviation of group L-Cv	0.402	0.0094	0.0078	0.0074
2	d) Standard test value H ₁	1.11	2.22	-0.25	2.88
	Heterogeneity measure H₂				
	a) Observed average of L-Cv/L-Skewness distance	0.1264	0.1041	0.1189	0.1165
	b) Simulated mean of of L-Cv/L-Skewness distance	0.0694	0.0911	0.0828	0.0776
3	c) Simulated standard deviation of of L-Cv/L-Skewness distance	0.0162	0.0167	0.0137	0.0121
	d) Standard test value H ₂	3.51	1.62	2.61	2.11
	Heterogeneity measure H₃				
	a) Observed average of L-Skewness /L-Kurtosis distance	0.1495	0.1094	0.1338	0.1143
	b) Simulated mean of of L-Skewness /L-Kurtosis distance	0.0788	0.0885	0.0973	0.1089
	c) Simulated standard deviation of L-Skewness /L-Kurtosis distance	0.0176	0.0187	0.0158	0.0145
	d) Standard test value H ₃	3.9	1.12	2.3	0.38

Regional flood frequency distribution

Several methods are available for selecting appropriate regional distributions. In this study the choice of regional frequency distribution was based on the results of L-moment ratios as described by Hosking and Wallis (1997). Additionally probability plots (plotting position) were used to verify that the selected regional the selected regional distributions provided a satisfactory description of the observed AMS.

L-moment ratio diagrams

An L-moment ratio diagram of L-kurtosis versus L-skewness compares sample estimates of the dimensionless ratios with their population counterparts for a range of statistical distributions include General Logistic (GLO), General Extreme Value (GEV), General Normal (GNO), Pearson Type 3 (PE3) and General Pareto (GPA). L-moment diagrams are useful for discerning grouping of sites with similar flood frequency behavior, and identifying the statistical distribution likely to adequately describe this behavior. Figure 2 and 3 shows the L-moment ratio diagram for homogeneous regions in west Azarbaijan province (A, B, C and D).

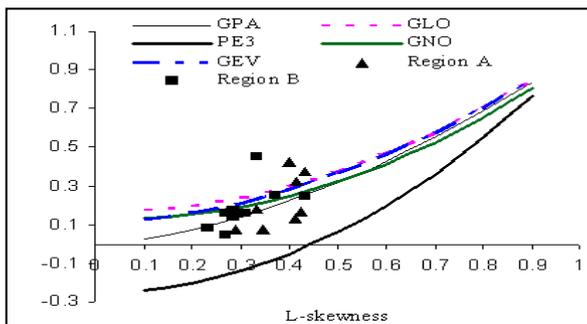


Figure 2. L-moment ratio diagram for A and B regions

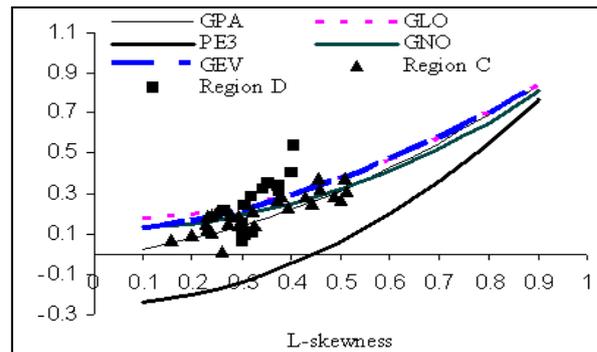


Figure 3. L-moment ratio diagram for C and D regions

As the sample L-moments, are unbiased, the sample points should be distributed above and below the theoretical line of a suitable distribution. From the above L-moment diagrams, it appears that the GPA and GEV distributions for region A, the GPA and GNO for region B, the GNO, GEV and GLO distributions for region C and the GLO, GNO, GEV and GPA distributions for region D are appropriate.

Goodness-of-fit test

The goodness-of-fit test described by Hosking and Wallis (1997) is based on a comparison between sample L-kurtosis and population L-kurtosis for different distributions. The test statistic is termed Z^{DIST} and given as follow

$$Z^{DIST} = (\tau_4^{DIST} - t_4^R + B_4) / \sigma_4 \quad (20)$$

Where DIST refer to the candidate distribution. τ_4^{DIST} is the population L-kurtosis of selected distribution, t_4^R is the regional average sample L-kurtosis, and σ_4 is the standard deviation of regional average sample L-kurtosis. The kappa distribution was

used to simulating 500 regions similar to the observed region. Then from these simulations, B_4 and σ_4 were estimated. Declare the fit to be adequate if Z^{DIST} is sufficiently close to zero, a reasonable criterion for selection of suitable being $|Z^{DIST}| \leq 1.64$. The test described above applied to the four homogeneous region. For each region, the data were tested against the GLO, GPA, GEV, GNO and PE3 distribution. Table 4 shows the results.

Table 4. Test statistic Z^{DIST} of regional distribution

Distribution	Region A	Region B	Region C	Region D
GLO	4.98	2.13	2.65	2.43
GEV	3.33	0.39	1.03	1.08
GNO	2.80	0.04	0.14	-0.40
PE3	1.78	-0.73	-1.43	-2.93
GPA	-0.56	-3.54	-3.10	-2.89

Plotting position

As pointed out by Hosking et al. (1985), comparison of different regional frequency distributions against observed data cannot be used to discriminate between different distributions, as the observed data represents only one of an infinite number of realizations of the 'true' underlying population. However, the probability plots may reveal tendencies such as systematic regional bias in the estimation of the extreme events. To assess how well the proposed regional frequency distribution fit to the observed AMS, the calculated X_T -T relationships from Equation 21 for the some stations in four regions are shown in Figure 4 to Figure 7. The empirical exceedance probability for the ordered observations $X_{(i)}$ were calculated using the median probability plotting position as described by Hosking and show below

$$P[X > x_{(i)}] = i - .35/n \quad (21)$$

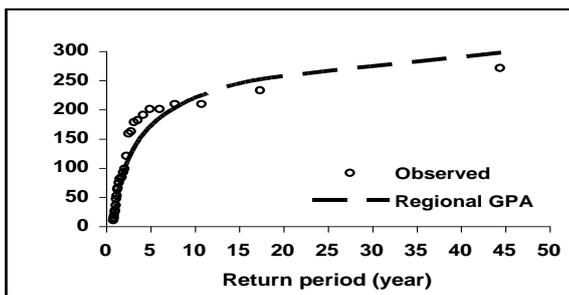


Figure 4. Probability plot for Baroon stations (region A)

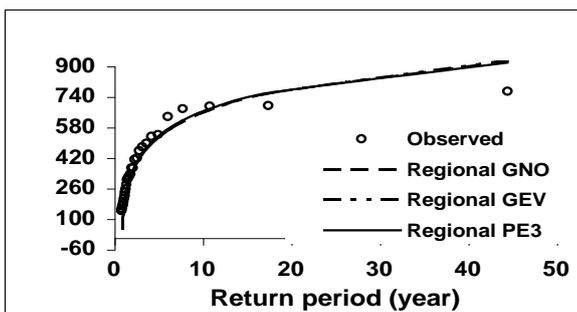


Figure 5. Probability plot for Kergal station (region B)

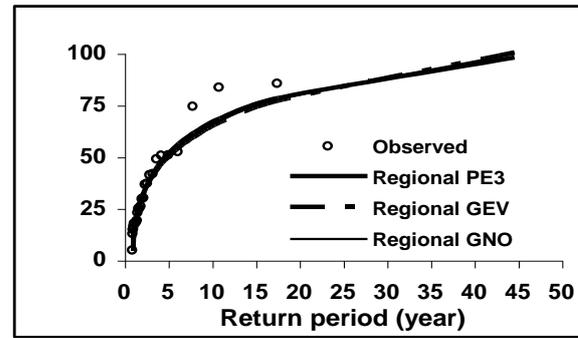


Figure 6. Probability plot for Alasagal station (region C)

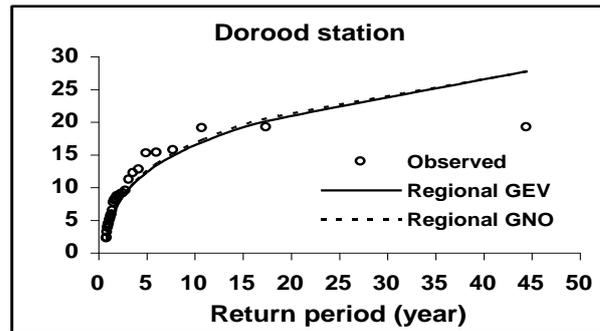


Figure 7. Probability plot for Dorud station (region D)

From above figures in region A and D the estimated X_T -T relationship give a good fit with the observed extreme events compared with the estimated X_T -T relationship in region B and C. From above three methods, goodness-of-fit test, L-moment ratio diagram and plotting position, the GPA distribution for region A, the GNO, GEV, PE3 distributions for region B and C and the GNO and GEV distributions for region D were selected as regional frequency distributions.

Quantiles estimation

After the regional distributions selected, using these distributions the quantiles with different nonexceedance probability estimated for regions A, B, C and D in west azarbayjan province. Table.5. shows the estimated value using GPA distribution in region A.

The accuracy of estimated values (regional and at-site estimations) determinate using relative root means square error (rRMSE). Figure 8 shows the rRMSE in some stations. Figure 8 shows that the rRMSE value in high return period is low. This indicates that both At-site and regional estimation procedure in high return period give accurate results.

In this models Q_p , A, C and Id are average AMS, basins area, runoff coefficient and design storm intensity respectively.

Regional model

Different methods are available for obtain regional models with hydrologic and basins parameters which can estimate AMS values in unengaged regions. In this study we obtained the regional models for regions A, B, C and D in west azarbayjan province using multiple regression and stepwise method. Table 6 shows the regional model for azarbayjan province basins.

Table 5. Estimated discharge (m3/s) from GPA distribution in region A

Station name	Nonexceedence probability						
	0.5	0.9	0.95	0.99	0.995	0.998	0.999
Gurool olya	153	277.3	299.6	323.5	328.3	331.5	333.1
Badlan	27.8	77.3	103.9	180.4	221.2	284.3	339.6
Baroon	95.1	244.9	254.1	293	302.7	310.3	313.5
Badavi	24.1	71.3	93.3	150.4	177.5	215.5	246.4
Makoo	125.1	250.3	272.9	298.2	302.2	306.2	307.5
Gale joog	89.9	224.2	260.1	310.9	324.6	337.3	343.7
Gara kurpi	46.3	128.9	164.1	244.9	278.8	323.9	357.2
Bashkand	21.3	46.2	53.0	63.3	66.2	68.9	70.4

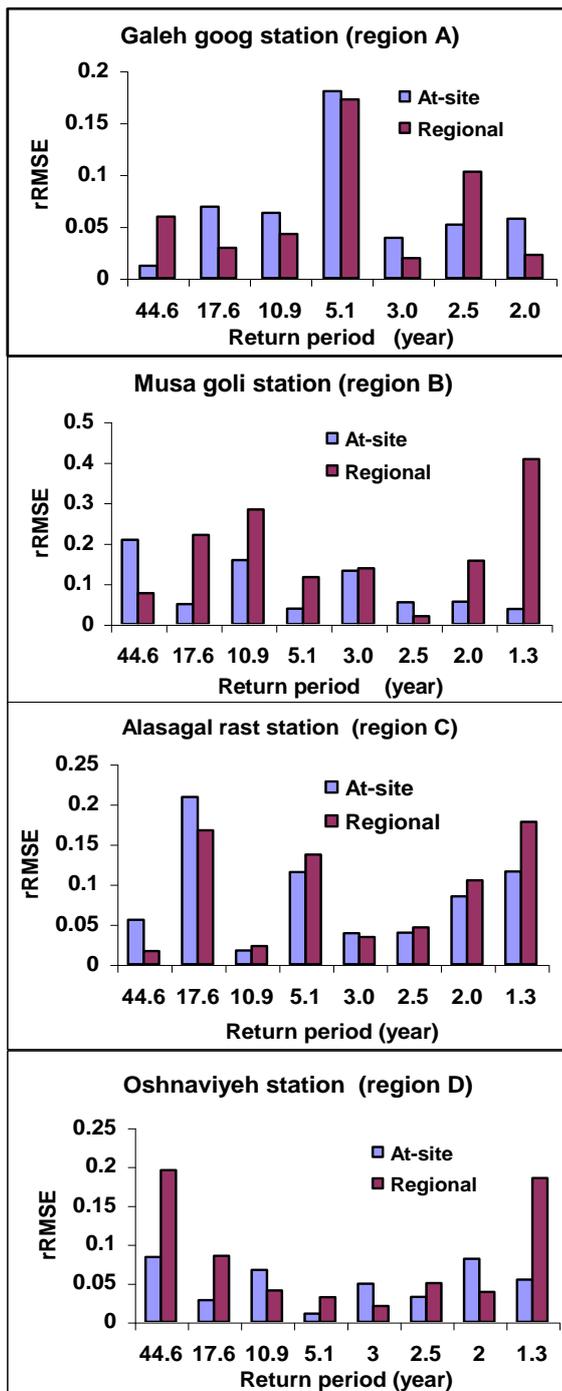


Figure 8. rRMSE between computed and observed data

Table 6. Regional models for studied regions.

Region	Regional Models	R ²
A	$Q_p = -0.0012(A \cdot C)^2 + 0.83(A \cdot C) + 15.59$	0.95
B	$Q_p = 0.2696(A \cdot C) + 17.853$	0.99
C	$Q_p = -6 \times 10^{-5}(A \cdot C \cdot I_d)^2 + 0.36(A \cdot C \cdot I_d) - 6.41$	0.93
D	$Q_p = -0.0004(A \cdot C)^2 + 0.51(A \cdot C) - 1.24$	0.95

CONCLUSION

In this study using site characteristic and Ward's method, hierarchical clustering method based on minimizing the Euclidean distance in site characteristics space within each cluster, the west azarbaijan province divided into four acceptably homogeneous regions. The heterogeneity measures based on H1 were 1.11, 2.22, -0.25, 2.88 for regions A, B, C and D respectively. The GPA distribution for region A, PE3, GNO and GEV distributions for region B and C and GEV and GNO distributions for region D were suitable and selected. The rRMSE values between computed and observed data were obtained. These values in high return period were low and indicate that both At-site and regional estimation procedure in high return period give accurate results. Regional models for homogeneous regions was obtained using the multiple regression and stepwise method and with catchment and hydrologic characteristics.

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