2013, Scienceline Publication

Journal of Civil Engineering and Urbanism

Volume 3, Issue 4: 216-220 (2013) (Received: June 03, 2013; Accepted: July 17, 2013; Published: July 30, 2013) ISSN-2252-0430

An Assessment of a Modal Pushover Method Based On 2DOF Modal System for Tall Asymmetric-Plan Buildings

Seyyed Mohsen Seyyedi Viand[1*], Kazem Shakeri[2], Hossein Vakili[1] and Mehrdad Azadi Hir[3]

[1]*Master of Civil Engineering, Department of Civil Engineering, University of Mohaghegh Ardabili, Ardabil, Iran.*
[2]*Assistant professor, Department of Civil Engineering, University of Mohaghegh Ardabili, Ardabil, Iran.*
[3]*Department of Civil Engineering, Germi Branch, Islamic Azad University, Germi, Iran.*

*Corresponding author's Email address: msv1561@yahoo.com

ORIGINAL ARTICLE

ABSTRACT: Recently, a novel two-degrees-of-freedom (2DOF) modal system has been proposed by Lin and Tsai which considers bifurcating and interaction between modal translation and modal rotation. Also, they claim that the proposed system is accurate than the conventional single-degree-of-freedom (SDOF) modal system in estimating the seismic demands of asymmetric-plan buildings. In this paper, the efficiency of application of the proposed system for tall buildings is evaluated. Hence, a modal pushover analysis is performed for a sample tall asymmetric-plan building based on SDOF and 2DOF modal systems. Then, the seismic responses of building are estimated by utilizing nonlinear time history analysis (NTHA). Finally, the analytical results are compared. Results show that the efficiency of 2DOF modal system becomes considerable at high levels of building, in particular for rotational responses. Consequently, the new system may be an alternative to the SDOF modal system in estimating the seismic responses of asymmetric-plan buildings.
Keywords: Asymmetric-Plan Buildings, Two-Degrees-Of-Freedom Modal Systems, Modal Pushover Analysis

INTRODUCTION

In 2006, Lin and Tsai showed that a two-degrees-of-freedom (2DOF) modal system separates each generalized modal coordinate into two coordinates, i.e., the modal translation and the modal rotation. Where, the SDOF modal systems are not capable to separate these two coordinates. When centre of stiffness (CR) and centre of mass (CM) are not coincident only in the x-axial direction, i.e., only u_z and u_θ are coupled. Hence, the equation of motion for an N-storey one-way asymmetric-plan building with each floor simulated as a rigid diaphragm like Figure 1 is:

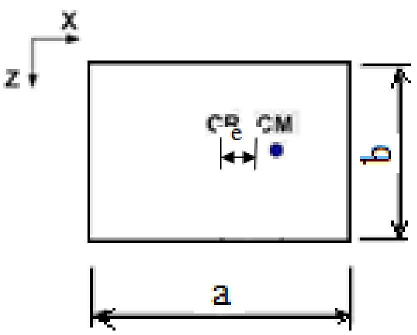

Figure 1. The floor plan of a one-way asymmetric-plan building

$$M\ddot{u}+C\dot{u}+Ku=-M\iota\ddot{u}_g(t)=-\sum_{n=1}^{2N}S_n\ddot{u}_g(t) \qquad (1)$$

The displacement vector, u, mass matrix, M, stiffness matrix, K, influence vector, ɩ, and modal participating factor, Γ, are simplified as [1]:

$$M=\begin{bmatrix} m & 0 \\ 0 & I_0 \end{bmatrix}_{2N\times 2N} \quad K=\begin{bmatrix} K_{zz} & K_{z\theta} \\ K_{\theta z} & K_{\theta\theta} \end{bmatrix}_{2N\times 2N} \qquad (2)$$

$$\Gamma_{zn}=\frac{\varphi_n^T M\iota_z}{\varphi_n^T M\varphi_n} \quad u=\begin{bmatrix} u_z \\ u_\theta \end{bmatrix}_{2N\times 1} \quad \iota=\begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2N\times 1}$$

The transformations of roof translation, $u_{zn,r}$, roof rotation, $u_{\theta n,r}$, base shear, V_{bn} and base torque, T_{bn} are

$$D_{zn}=\frac{u_{zn,r}}{\Gamma_{zn}\varphi_{zn,r}} \qquad D_{\theta n}=\frac{u_{\theta n,r}}{\Gamma_{zn}\varphi_{\theta n,r}} \qquad (3)$$

$$A_{zn}=\frac{V_{bn}}{\Gamma_{zn}^2 M_n} \qquad A_{\theta n}=\frac{T_{bn}}{\Gamma_{zn}\Gamma_{\theta n}M_n} \qquad (4)$$

Where:

$$\Gamma_{zn}=\frac{\varphi_n^T M\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\varphi_n^T M\varphi_n} \qquad \Gamma_{\theta n}=\frac{\varphi_n^T M\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\varphi_n^T M\varphi_n} \qquad (5)$$

$$M_n=\varphi_n^T M\varphi_n$$

Considering Figure 2, the separation of two coordinates is allocated to the nonlinear phase:

To cite this paper: S. M.Seyyedi Viand, K. Shakeri, H.Vakili and M. Azadi Hir. 2013. An Assessment of a Modal Pushover Method Based On 2DOF Modal System for Tall Asymmetric-Plan Buildings. *J. Civil Eng. Urban.* 3 (4): 216-220.
Journal homepage http://www.ojceu.ir/main/

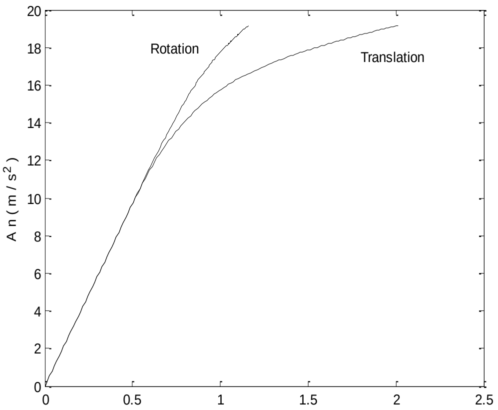

Figure 2. The typical pushover curves representing the roof translation versus base shear and roof rotation versus base torque relationships in ADRS format

2DOF modal system

A 2DOF modal stick includes a rigid beam and a rigid column. The beam is connected with the column by a rotational spring whose stiffness is $K_{\theta n}$. The length of the column is equal to one and connected with the ground by a rotational spring whose stiffness is K_{zn}. The mass, m, and moment of inertia, I, are concentrated at the end of beam as shown in Figure 3.

There are only two possible DOFs which are shown in the Figure 3 as the Z-directional translation and y-directional rotation at CM. The five elastic parameters of this system can be defined as [1]:

$$m=\varphi_{zn}^{T} m \varphi_{zn} \quad e=\frac{\varphi_{zn}^{T} K_{z\theta} \phi_{\theta n}}{\varphi_{zn}^{T} K_{zz} \varphi_{zn}} \quad I=\varphi_{\theta n}^{T} I_{0} \varphi_{\theta n}$$

$$K_{z}=\varphi_{zn}^{T} K_{zz} \varphi_{zn} \quad K_{\theta}=\varphi_{\theta n}^{T} K_{\theta\theta} \varphi_{\theta n}-\frac{(\varphi_{zn} K_{z\theta} \varphi_{\theta n})^{2}}{\varphi_{zn}^{T} K_{zz} \varphi_{zn}} \tag{6}$$

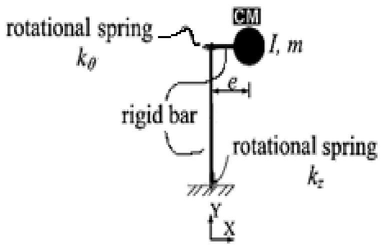

Figure 3. A 2DOF modal system

The modal inertia force for the 2DOF system is as

$$s=\Gamma \tilde{M} \varphi=1 \cdot \begin{bmatrix} m_{n} & 0 \\ 0 & I_{n} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}=\begin{bmatrix} m_{n} \\ I_{n} \end{bmatrix} \tag{7}$$

By exciting the original building by the n^{th} modal inertia force, the elastic translational and rotational deformations of the system can be defined as [1]:

$$\tilde{D}_{zn}=\frac{m_{n}}{K_{zn}}-\frac{(I_{n}-m_{n} e_{n}) e_{n}}{K_{\theta n}} \tag{8}$$

$$\tilde{D}_{\theta n}=\frac{I_{n}-m_{n} e_{n}}{K_{\theta n}} \tag{9}$$

$$\tilde{D}_{zn}=\tilde{D}_{\theta n} \tag{10}$$

Setting the right hand side of equation (8) equal to that of equation (9) yields

$$\frac{K_{\theta n}}{K_{zn}}=(\frac{I_{n}}{m_{n}}-e_{n})(1+e_{n}) \tag{11}$$

Inelastic parameters of 2dof modal system

Before calculating the inelastic parameters of 2DOF modal system, some key parameters should be defined, i.e., post-yielding stiffness ratios, α_{zn} and $\alpha_{\theta n}$, and yielding force. In this regard, the pushover curves of n^{th} mode should be idealized as shown in Figure 4. In asymmetric-plan buildings, the translational yield should be equal to the rotational yield

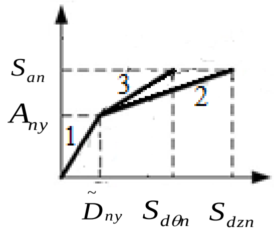

Figure 4. The two bilinear pushover curves for one-way asymmetric-plan structure

$A_{zny}=A_{\theta ny}=A_{ny}$. Since the modal translation, $\tilde{D}_{zn}$, is equal to the modal rotation, $\tilde{D}_{\theta n}$, for elastic 2DOF modal systems, the stated assumption also implies that the yielding modal translation equal to the yielding modal rotation, or $\tilde{D}_{zn}=\tilde{D}_{\theta n}=\tilde{D}_{ny}$ as shown in Figure 4. The slopes of lines 1, 2 and 3 are equal to ω_n^2, $\alpha_{zn}\omega_n^2$ and $\alpha_{\theta n}\omega_n^2$, respectively. By determining the four intended values, inelastic parameters of the 2DOF modal system can be defined as [1]:

$$K'_{zn}=\frac{m_{n}}{\dfrac{\dfrac{m_{n}}{K_{zn}}-\dfrac{(I_{n}-m_{n} e_{n}) e_{n}}{K_{\theta n}}}{\alpha_{zn}}+\dfrac{(I_{n}-m_{n} e_{n}) e_{n}}{K'_{\theta n}}}$$

$$K'_{\theta n}=K_{\theta n} . \alpha_{\theta n} \tag{12}$$

$$M_{y\theta n}=A_{\theta ny} I_{n}-A_{zny} m_{n} e_{n} \qquad M_{yzn}=A_{zny} m_{n} \tag{13}$$

Where, M_{yzn}, $M_{y\theta n}$ and K'_{zn}, $K'_{\theta n}$ are the yielding moments and the post-yielding stiffness of the two rotational springs of the 2DOF modal system. Thus, the total elastic and inelastic parameters of the 2DOF modal system are obtained by using equations (6), (12) and (13).

ANALYTICAL EXAMPLE

The selected structural model is a nine storey steel structure from SAC buildings [2]. In order to achieve an irregular one-way plan building, CM is intentionally placed with 20% eccentricity with respect to the centre of the floor plan. The sides of the structure are denoted as stiff side and flexible side, taking into account the position of the centre of mass and the initial centre of stiffness as shown in Figure 5.

The mass and the mass moment of inertia for each floor are shown in Table 1. Also, the main properties of

To cite this paper: S. M.Seyyedi Viand, K. Shakeri, H.Vakili and M. Azadi Hir. 2013. An Assessment of a Modal Pushover Method Based On 2DOF Modal System for Tall Asymmetric-Plan Buildings. *J. Civil Eng. Urban.* 3 (4): 216-220.
Journal homepage http://www.ojceu.ir/main/

the records acceleration time-histories selected for the dynamic analysis are briefly reported in Table 2.

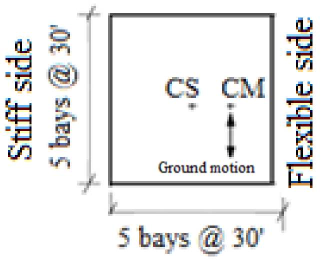

Figure 5. Plan of sample asymmetric-plan building

Table 1. Properties of selected building.

Mass (lb)		**Mass moment of inertia(lb.ft^2)**	
Floor2	226507	Floor2	1053265259
Floor3-9	222636	Floor3-9	1035263028
Roof	239827	Roof	1115203778

Table 2. Ground motions properties.

Earthquake	**Year**	**Station**	**PGA(g)**
Northridge	1994	77Rinaldi	0.83
Loma Prieta	1989	16LGPC	0.56
Landers	1992	24Luceme	0.72
Kobe	1995	0KJMA	0.82
Erzican	1992	95Erzican	0.51
Tabas	1978	9101Tabas	0.83
Duzce	1999	LDEO-0375VO	0.53

The proportional damping used in the analysis of the prototype building and modal sticks is equal to 5%. The plan of building is shown in Figure 5. The period of vibration and dominant motion of each mode for this asymmetric building are listed in Table 3. For abbreviation, only first 10 modes are shown in the table. All the modelling and analyzing processes were carried out using OPENSEES computer program which can provide the feature of inputting ground acceleration records not only in translation, but also in rotation.

Step by step nonlinear analysis for sample building

- Perform the eigenvalue analysis of the original N-storey asymmetric-plan building. Compute the elastic properties of the building, including the vibration periods of modes, the modes shapes, the modal participation masses and dominant motion of each mode. Considering three directions of motion (z, x and θ), there are 27 modes for a nine storey building in a 3D problem. The elastic properties of the example building are shown in Table 3. It shows that the accumulation of the modal participation mass up to 7th mode is over 90% regarding the topmost amount among all three directions. Therefore, only the 1st to the 7th modes are selected in this analytical example. It should be noticed that only u_z and u_θ are coupled. So, those modes whose X-directional motions are dominant will be eliminated in calculating the total responses.
- Apply equation (6) and compute the elastic properties for each 2DOF modal sticks.
- Apply equations (3) and (4) to conduct the nth pushover curves by applying the S^*_n defined in equation (14):

$$S_n^* = \begin{Bmatrix} m\varphi_z \\ I\varphi_\theta \end{Bmatrix} \qquad (14)$$

- Idealize the pushover curves obtained from step 3 as bi-linear curves. Apply equations (12) and (13) to compute the inelastic properties for each 2DOF modal stick. An important question which arises here is that between two idealized curves in each mode, which of them should be chosen? The appropriate curve in each mode is selected based on Table 3. The maximum modal participation mass determines direction of dominant motion for each mode. Thus, in each mode, that curve whose direction of motion is dominant is selected. For example, the dominant motion of the 1st mode is Z-directional.
  So, in this mode, to determine the inelastic parameters of 2DOF modal stick the translational bifurcated curve is used.
- Conduct the 2DOF modal stick regarding the elastic and inelastic parameters which have been specified in steps 2 and 4.
- Calculate the maximum inelastic response of 2DOF modal system in the nth mode by means of time history analysis. The vibration period of nth mode can be presented as [1]:

$$T_n = \frac{T_{zn}}{\sqrt{1+e_n}} \qquad (15)$$

  Where, T_{zn} is the translational vibration period of corresponding uncoupled 2DOF modal system.
- Compute the maximum roof translation and maximum roof rotation of the nth mode in direction of dominant motion.
- Determine the desired responses (displacement, rotation, storey drifts at the centre of mass and in the flexible or stiff side of building) when the calculated responses are equal to those obtained from step 7.
- Determine the total responses (demands) by combining gravity responses and the peak the modal responses using the "SRSS" rule.

Table 3. Properties of each mode of prototype building.

Mode no.		1	2	3	4	5	6	7	8	9	10	…	…
Period(s)		2.55	2.26	1.42	0.93	0.84	0.54	0.52	0.48	0.35	0.31	…	…
Dominant motion		Z	X	θ	Z	X	Z	θ	X	Z	X	…	…
$\frac{\Gamma_n^2 M_n}{\sum_{i=1}^{9}\Gamma_i^2 M_i}(\%)$	X	0.00	82.10	0.00	0.00	11.17	0.00	0.00	4.03	0.00	1.44	…	…
	Z	78.72	0.00	4.35	10.30	0.00	2.93	1.12	0.00	1.18	0.00	…	…
	θ	4.43	0.00	79.02	0.50	0.00	0.94	9.44	0.00	0.11	0.00	…	…

To cite this paper: S. M.Seyyedi Viand, K. Shakeri, H.Vakili and M. Azadi Hir. 2013. An Assessment of a Modal Pushover Method Based On 2DOF Modal System for Tall Asymmetric-Plan Buildings. *J. Civil Eng. Urban.* *3* (4): 216-220.
Journal homepage http://www.ojceu.ir/main/

In this section, the figures obtained from analysis performed on the sample building are displayed. These figures indicate total errors of using SDOF and 2DOF modal systems in estimating the seismic responses of asymmetric-plan structures. Each figure is derived from a mean of applying 7 records on the sample building. These responses are calculated in three significant parts of floor plan: Centre of mass, Flexible side and Stiff side of plan. Formulas used to calculate the total errors of performing nonlinear analysis on the sample n-story building are given as [3]:

Total error index in calculating the stories displacement and rotation

$$Error_{Dis}(\%) = 100 \times \frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(\frac{Dis_{i-NTHA} - Dis_{i-Push}}{Dis_{i-NTHA}}\right)^2} \quad (16)$$

$$Error_{Rot}(\%) = 100 \times \frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(\frac{Rot_{i-NTHA} - Rot_{i-Push}}{Rot_{i-NTHA}}\right)^2} \quad (17)$$

Where, $Dis_{i\text{-}NTHA}$ and $Rot_{i\text{-}NTHA}$ are the peak translation and peak rotation of the i^{th} storey due to the nonlinear dynamic analysis, respectively. Also, $Dis_{i\text{-}Push}$ and $Rot_{i\text{-}Push}$ are the maximum translation and maximum rotation of the i^{th} storey due to pushover analysis.

Total error index in calculating the stories drift

$$Error_{\Delta}(\%) = 100 \times \frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(\frac{\Delta_{i-NTHA} - \Delta_{i-Push}}{\Delta_{i-NTHA}}\right)^2} \quad (18)$$

Where, $\Delta_{i\text{-}NTHA}$ is the maximum drift of i^{th} storey due to the nonlinear dynamic analysis and $\Delta_{i\text{-}Push}$ is the maximum drift of i^{th} storey due to pushover analysis and n is the number of stories.

SUMMARY AND CONCLUSION OF EVALUATION

Considering Figure 6 up to Figure 10, it can be mentioned that the accuracy of using proposed system in estimating the seismic responses of asymmetric-plan buildings is good. This accuracy becomes considerable by increasing levels of building. Because, by increasing the levels of building, the rotational motions become dominant in lower modes. Therefore, considering the interaction between translation and rotation by using the 2DOF modal system, can model the seismic performance of building more realistic than SDOF modal system. Since the importance of rotational responses in designing purposes of tall buildings, utilizing 2DOF modal system can be considered for asymmetric-plan buildings.

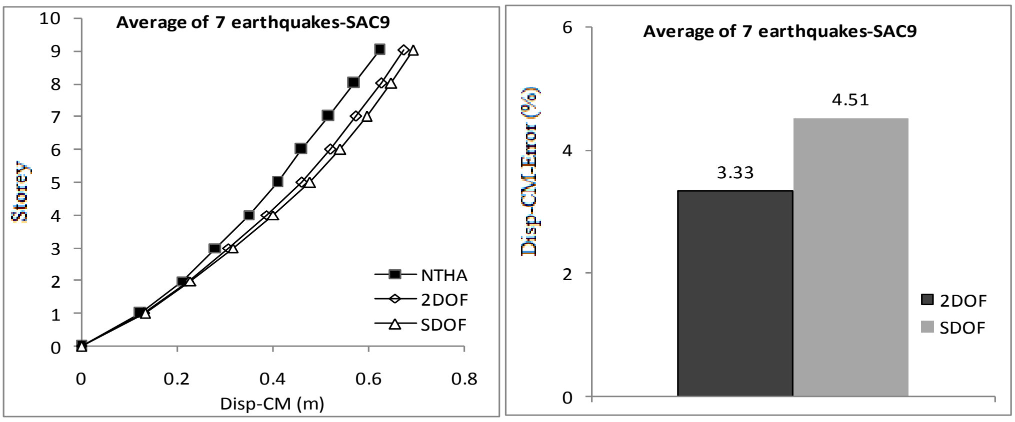

Figure 6. Average of responses profiles resulting from NTHA and pushover analysis based on SDOF and 2DOF modal systems, (a) stories displacement at CM, (b) Total errors of the stories displacement.

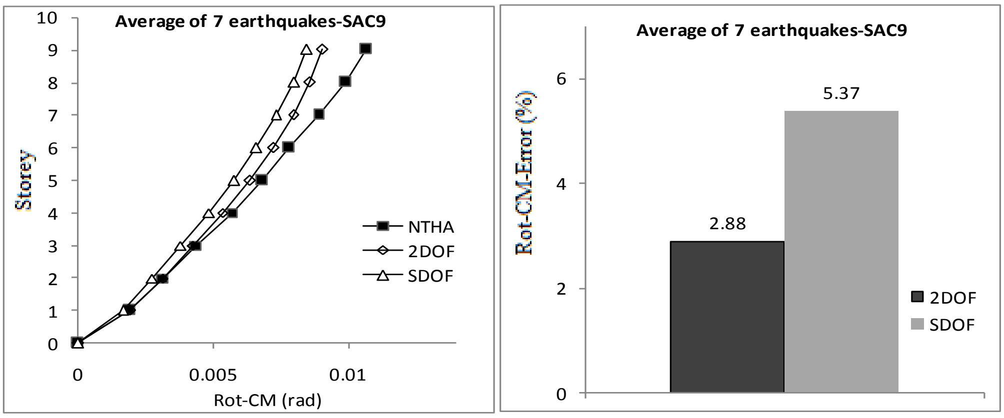

Figure 7. Average of responses profiles resulting from NTHA and pushover analysis based on SDOF and 2DOF modal systems, (a) stories rotation at CM, (b) Total errors of the stories rotation.

To cite this paper: S. M.Seyyedi Viand, K. Shakeri, H.Vakili and M. Azadi Hir. 2013. An Assessment of a Modal Pushover Method Based On 2DOF Modal System for Tall Asymmetric-Plan Buildings. *J. Civil Eng. Urban. 3* (4): 216-220.
Journal homepage http://www.ojceu.ir/main/

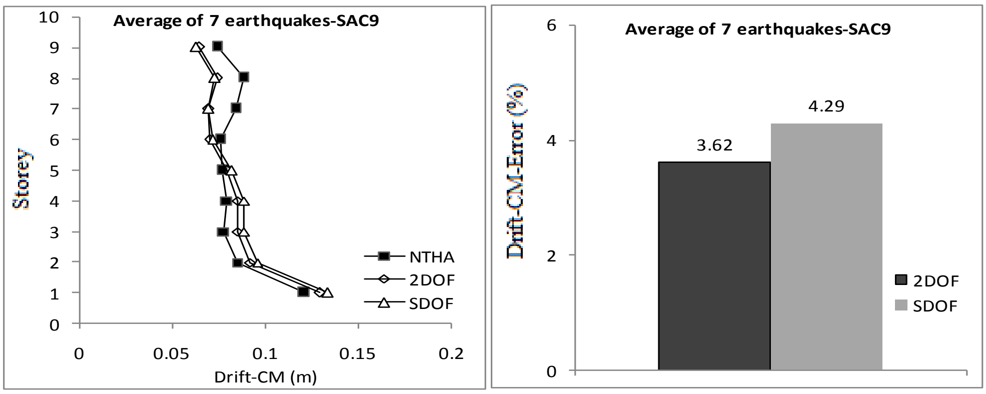

Figure 8. Average of responses profiles resulting from NTHA and pushover analysis based on SDOF and 2DOF modal systems, (a) stories drift at CM, (b) Total errors of the stories drift at CM.

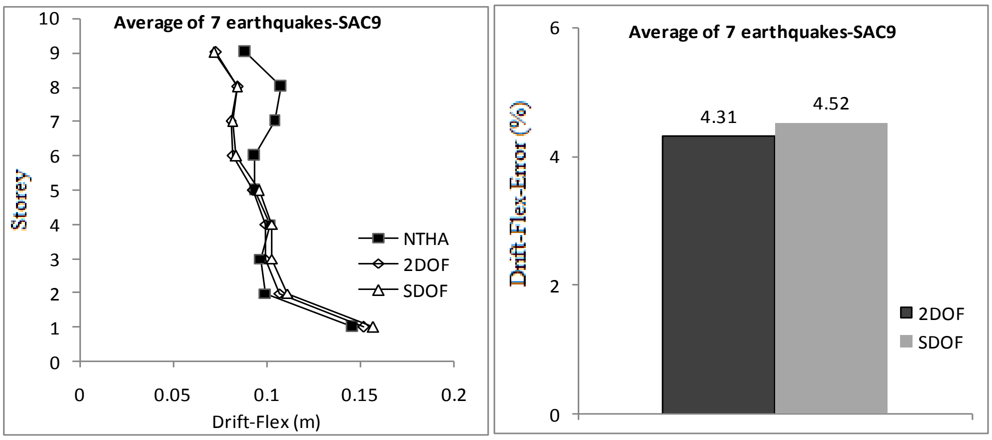

Figure 9. Average of responses profiles resulting from NTHA and pushover analysis based on SDOF and 2DOF modal systems, (a) stories drift at flexible side of floors, (b) Total errors of the stories drift at flexible side.

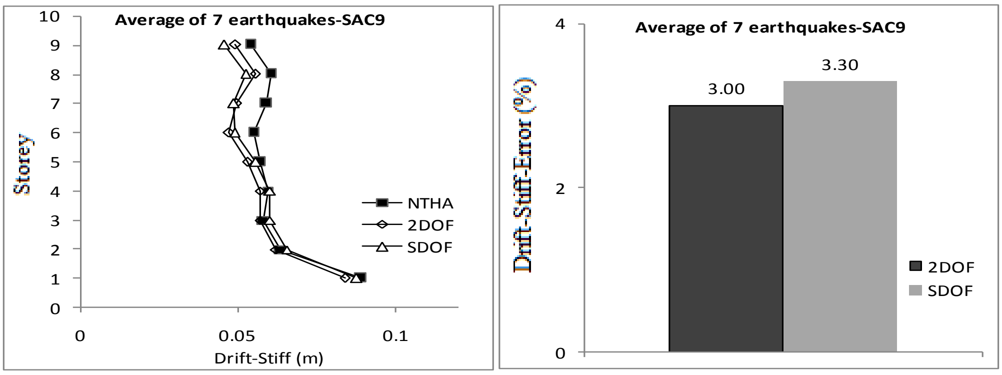

Figure 10. Average of responses profiles resulting from NTHA and pushover analysis based on SDOF and 2DOF modal systems, (a) stories drift at stiff side of floors, (b) Total errors of the stories drift at stiff side.

REFERENCES

Dynamics of structures. Theory and applications to earthquake engineering translated by S.Tahooni. Published by Elm va Adab publications (1387).

Gupta, A. and Krawinkler, H. (1999). Seismic Demands for Performance Evaluation of Steel Moment Resisting Frame Structures. The John A. Blume Earthquake Engineering Center. Report No.132.

Lin, J.L. and Tsai, K.Ch. (2006). Simplified seismic analysis of asymmetric building systems. Earthquake Engineering and Structural Dynamics. 36: 459-479.

Lin, J.L. and Tsai, K.Ch. (2009). Modal Parameters for the Analysis of Inelastic Asymmetric-Plan Structures. Earthquake Spectra. 25(4), 821-849.

Lopz-Menjivar and Pinho, R. (2004). A review of existing pushover methods for 2D reinforced concrete buildings. Pavia (Italy): Rose school