

# The Influence of Material Properties and the Geometrical Parameters of the Operating Arch on the Stability of Vaults

Nima Valibeig<sup>1\*</sup>, Mozhgan Esmacili<sup>1</sup>, Atefeh Ansari<sup>1</sup>

Faculty of Conservation and Restoration, Art University, Isfahan, 1744, Iran

\*Corresponding author's E-mail: N.valibeig@au.ac.ir

**ABSTRACT:** One of the key elements in architecture is vault. Because timbers are locally available only in certain parts of Iran, the vaults used to be built in various arched forms especially in traditional buildings. Identifying the structural characteristics of such vaults may reveal the innovative approaches adopted by the Persian architects who tried to design and build the vaults in based on a sustainable architecture; a kind of architecture that was, from one hand, adaptable to climate, location and local materials, and capable, from the other hand, to meet the users' needs. The materials used in and the geometric characteristics of the arches are two determining factors in vault stability. The major part of the past research has focused on examining the strength of the arches statically. The present study, however, aimed to analyze the impact of materials and geometric characteristics on the vault resistance. To this end, the behavior of geometric arches in different forms and sizes along with various materials were studied. The identified generate arches were drawn by design soft wares and then analyzed by resistance analyzers. The results clearly revealed the effective role of geometric parameters, materials and thickness of arch base in vault stability.

**Keywords:** Sustainable architecture; Stability; Generate arch; Geometry; Material.

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## INTRODUCTION

During the long years the Persian architects, thanks to their experience and innovation, have managed to create magnificent buildings using locally available materials such as adobe, mud, brick, stone, gypsum, and lime. These seemingly simple buildings are typical example in which structurally static rules have been fully respected (Imantalab, 2010). The architecture in question does care for environmental and climatic considerations and takes advantage of natural resources sensibly. This is the case-long before the emergence of challenging problems such as energy crises and environmental problems and along before the recognition of the idea of sustainable architecture. The vaults, as one of the load carrier and structurally key elements in historical buildings, are noteworthy here. The vaults, depending on their forms, have different bearing capacities. In the Persian architecture, a wide variety can be observed in built vaults in terms of forms and materials used.

These structures (vaults and domed) demonstrate several advantages in hot and dry climates, especially in areas with a wide range of diurnal temperatures. Given their inherent thermal mass and their greater heights at the center of a space, where light, warm air gathers and can be easily discharged through openings, vaulted spaces provide better natural climatic control (Minke, 2000).

*Chafd*, geometrically speaking, is a linear or curved form referring, in the Persian architecture, to the rib vault placed over the entrance door. *Chafd* is two-dimensional in form. However, if it extends alongside the wall it turns into a three-dimensional form creating a

*vault*. In the Persian architecture, the selection of appropriate *chafds* to resist the imposing loads on the vaults and domes, has been based on static and structural considerations (Pirnia, 1994). The examination of structural behavior of such vaults may offer helpful information not only to investigators in general, but more specifically to those involved in restoring historical buildings.

During the course of time, the various arches have been applied through different methods. The different arches demonstrate different limits of resistance against the imposing loads. Having done different studies, the authors recognized, examined and analyzed the most frequently used arches in the Persian architecture. The also managed to customized the Arco structural software, (a special software to measure the stability of vaults) for the first time, based on different versions of vernacular architecture. First, the Persian arches were modeled and then the results were analyzed. To this end, the arches were drawn carefully by the relevant software and then the corresponding coordinates of individual points were determined. The data were inputted into Arco software and modeled and analyzed afterwards. This software, developed by the structural researchers of Barry University, Italy, operates based on drawing thrust line. It should be noted that since Arco has been originally developed to analyze the vaults in western architecture, it is capable to model the semi-circle and Ogive arches. The customization process applied on this software, however, made it compatible with analyzing the Persian arches.

The key questions this study intends to answer are as follows:

1-Considering the variation in the stability of vaults, what is the optimum thickness for the Panj-O-Haft arch (Persian arch)?

2-Considering the principles of sustainable architecture, what is the influence of varied materials on the stability of vaults?

## MATERIALS AND METHODS

### Arco Software: A Brief History

The modern study of masonry arches starts with Heyman's work, through the application of the ideas of plasticity theory. The "safe theorem" can be stated as follow: a masonry arch is safe if a line of thrust, in equilibrium with external loads and lying wholly within the thickness, can be found and the corresponding stresses are sufficiently low (Gelfi and Capretti, 2001). Masonry vaults constructed of numerous blocks of material pressing against one another exert not only the accumulated downward weight of the material and of any superimposed load but also a lateral thrust, or tendency to spread outward (Borges, 2001).

In every structure, the individual structural constituents must be strong enough to resist the imposing internal actions of structure. To make sure that the vaults will not undergo tensile tensions or cracklings, materials such as brick, adobe or stone must be used in a way that the resultant of forces is focused on the middle one-third of the cross section (Millais, 2005).

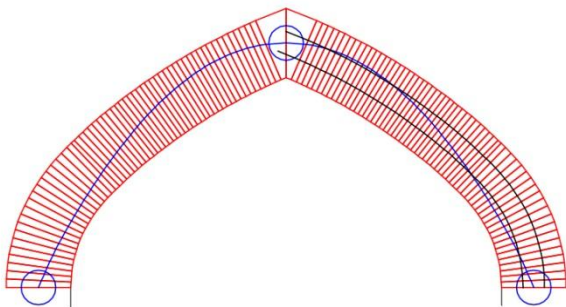


Figure 1. The thrust line of an arch

On this basis, Arco software was developed by professor Gelfi, Italy, in order to analyze the stability of arches based on thrust line. Using this software, precise structural analyses are obtainable through drawing the arches in different modes including inputting geometrical points for different western arches and modeling different loads. It is also capable to calculate the internal and external tensions, thrusts at support points as well as the varied imposed pressure values at different points of the arch. In order to test the reliability of the results obtained, several cases of built vaults with special arches were analyzed mathematically and tested also at laboratory. The software results and those of obtained from the calculations and laboratory tests were significantly comparable. The mentioned software has been widely referred to in the papers published on analysis of stability of vaults (Gelfi, 2002).

The needed factors to analyze the arches structurally through using this software include arch span, vault support thickness, vault rise,  $\gamma_m$  (the density

of materials used in the vault structure) and ring width. In the case of used filling materials in the vault structure, the specifications of materials are specified by  $\gamma_1$  and  $\gamma_2$  and  $h_1$  and  $h_2$  (Figure 2 and 3).

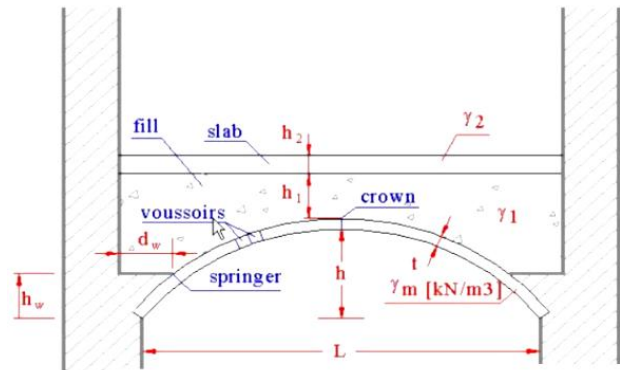


Figure 2. Different constituents of the arch in Arco software

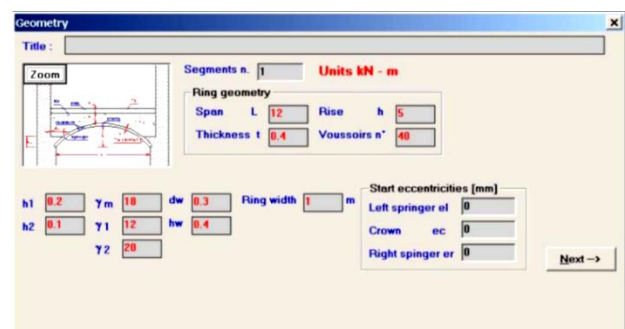


Figure 3. The needed factors to analyze the arches structurally through Arco software

### Sustainable Architecture

The first steam engine was invented in England in 1769. By this event, the Industrial Revolution and modern technology started to emerge followed, not too long after, by establishing the first railroad and building the first steam ship. The construction industries advanced too accordingly and materials such as steel beams and cement were produced massively during the 19<sup>th</sup> century. Too much factories were set up and urbanism started to expand (Ghobadian, 2008). Noticeable technological advances, valuable achievements and promising future brought about by modernity were just one side of the picture; the other side was quite dark. With the advent of Industrial Revolution and the shifted people's lifestyle, technology turned to be the plight of nature. The construction of buildings fueled the environmental crises (Ahmadi, 2012).

The energy crisis of the 1970s and the subsequent environmental problems cast shadow over the many of the human activities challenging architecture and urban planning- two fields that were especially responsible for their role in increased energy consumption and environmental pollutions. Hence, the idea of sustainability and the issue of protecting the earth ecologically as well as caring for the future generations were posed in the literature of many fields. The concept of sustainability in designing and developing built environment was examined in small and large scales.

Sustainability, which is based upon the three underlying principles, namely economically spending natural resources and designing based upon natural life cycles as well as humanistic values, brought about major changes in the fields of production and consumption. As a result, architecture and urbanism were directed towards making the most of environmental forces and being compatible with ecology (Hosseini et al., 2008).

The summit of controversies on the issue of energy crisis and sustainable development happened in the Earth Summit held in Rio de Janeiro in 1992 known afterwards as Rio Summit. In Rio Summit a resolution was adopted calling for sustainable development in all countries of the globe and they were obliged to comply with it. Ten years later, in 2002, another meeting was held in Johannesburg, South Africa, in which the high rank authorities including ministers and environmental experts attended. Johannesburg meeting focused especially on items already approved in Rio Summit and prepared the ground to make them workable worldwide. The key definition of sustainable development provided in Johannesburg meeting was as follows:

“Development that meets the needs of the present without compromising the ability of future generations to meet their own needs and take into account the environmental issues as well”.

Although different definitions of sustainable development have been given so far, the common essence of all of them is caring for the future generations and preserving the environment around the globe. The principles of sustainable development take into consideration such issues as culture; native characteristics as well as past experiences; taking advantage of renewable energies and avoiding non-renewable ones. According to The World Commission of the United Nations, sustainable development can be defined as: (Zandieh and Parvardinejad, 2011).

“Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs”.

Despite the universal mottoes promoted by sustainable development, focusing on the environment at its global scale, it basically recommends applying *local* approaches so as realistic and workable solutions can be put in use and natural (bio) diversities can be preserved as well. Sustainable development defends the idea of "Think globally, act locally" (Ahmadi, 2003). Since about one half of the world energy is consumed in construction activities and the existing buildings around the globe can be considered as major source of producing greenhouse gases (Aliabadi et al., 2012), the issue of sustainable architecture merits special attention.

Concerned with ecological and climatic considerations, sustainable architecture is designed and implemented based of taking effectively advantage of natural resources. In such architecture, the utmost effort is made to keep the adverse impacts of construction activities on the surrounding environment to a minimum. Based on the OECD Plan, sustainable buildings are those by which the least possible undesirable impacts are imposed on the nearby (built) environment as well as surrounding areas. Keeping this in mind, doing construction activities in environments must be done with respect to the available natural resources, to

economical consumption of non-renewable resources such as fossil fuels and to taking care for posterity (Ahmadi, 2012). Here, the point deserving special care is that all factors involving in human privacy must considered as an integrated whole. Architecture is able to make people aware of great economic and environmental value of different energies known as harmless, green, calm, etc. (Askari, Ghasempoor, 2012).

The principles of sustainable architecture may be summarized as: (Ahmadi, 2012)

1-Energy saving: a building must be built in a way that the consumption of fossil fuels is kept at a minimum.

2-Compatibility with climate: a building must be compatible with its climate and with available energy resources in the construction site.

3-Reduced use of new sources of construction materials: A building must be built in a way that the least amounts of construction materials are put into use so that the built structure can, itself, be used as a new source of construction materials to build new one(s).

4-To meet the residents; needs: To meet the physical and psychological needs of the residents is a must.

5-Compatibility with construction site (place): A building must not be in conflict with its local site.

6-Holistic view: All the principles of sustainable architecture must be integrated in a full process so as a healthy built environment emerges.

## RESULTS AND DISCUSSIONS

### Examination of Arches

Masonry, whether it is stone or brick, is the most durable form of the construction material. More ever, it is usually reasonably easy to handle and it is one of the most versatile structural form which has used for arches, vaults, walls and pillars from the very beginning of history. Masonry possesses very low tensile strength and shear resistance. The compression-resistance approach in the structural form of the historical structures (pillars, arches, walls, domes...) is primarily based on this fact (Nese Sener 2004).

The history of using arches in the Persian architecture dates back to Pre-Islamic era. In this era, vaults over the staircase in Choqa Zanbil (1265 B.C) and those over spaces in Haft Tappeh are in oval shape (Zarei, 2010). The application of arches in wide spans in form of dome, vault and chafd is observable in Sassanid era. All the identified structures of Sassanid era have vaults- either using rubbles or bricks. The form of vaults, regardless of materials used or the consequences resulted from arching method, has remained the same (Pope, 1977). Prime representations of wide vaults in Sassanid era can be observed in structures such as Taq-e-Gara, Taq-e-Kasra, Taq-e-Firouz Abad and Taq-e-Sarvestan.

The operating element of a vault can be defined as "the intersection of the vault body with a plane perpendicular to the bearing support". By this definition, it is possible to have several operating elements (Besenval, 198). Given that the stability and the safety of curved structures under a given loading condition is strongly dependent on the geometry of the structures and on the

mechanical characteristics of the constituent material (Valluzzi et al., 2001).

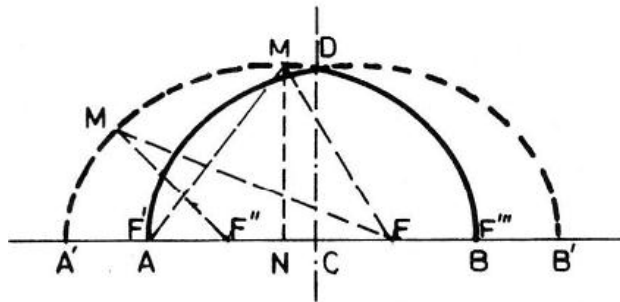


Figure 4. Drawing in oval method

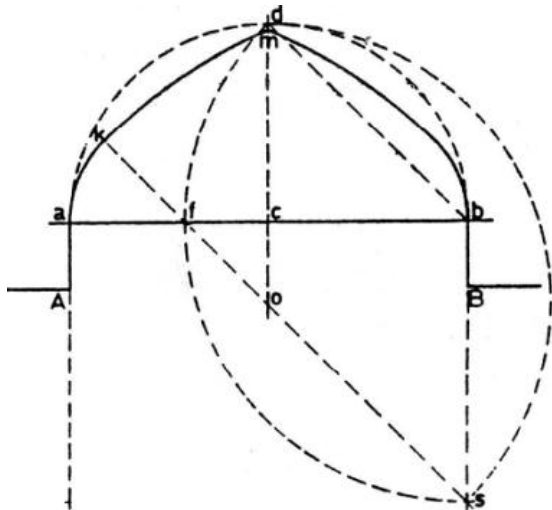


Figure 5. Drawing in compass method (Pirnia, 1994)

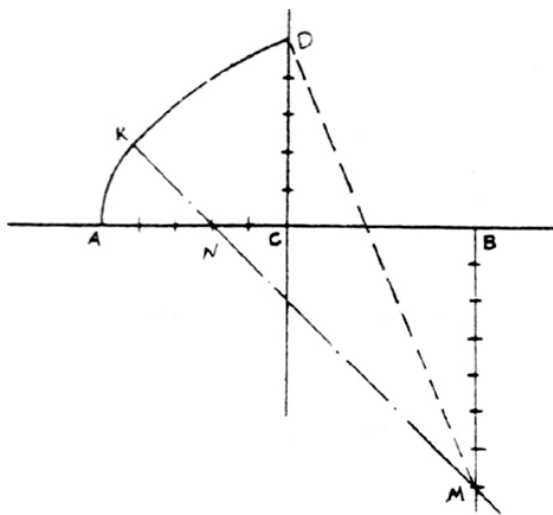


Figure 6. Drawing in numeric method (Pirnia, 1994)

The Panj-o-Haft arch, one of the most commonly used vaults in the Persian traditional architecture, was selected in the present study. The mentioned arch was drawn through three distinct methods namely, oval, compass and numeric methods (figures 4, 5 and 6). First, the arch in three modes, each with a 4-meter span, was drawn by AutoCAD software. Then, with the help of Arco software, the arch with different thicknesses (02, 04 and 06 meters) and with different materials (adobe, stone and brick) was analyzed.

#### Arco-based analysis of arches with varying thickness

In the Persian architecture, arches used to be built with different spans and thicknesses. The thickness of the arches must be large enough to keep the tensile tensions at a minimum. It follows that if the thickness is less than a specific threshold, the arch collapses (Morgan, 1978).

#### Analysis of the Panj-o-Haft arch in oval method:

##### Vault specifications

Span: 4 meters

Thickness: varying

Depth: 5 meters

Density of bricks:  $15 \text{ kN/m}^3$

The results revealed that as the thickness decreases from 0.6 to 0.4 meter, the maximum tension in the external parts and in the vault abutment decreases too. However, as it decreases to 0.2 meter, the tension again increases. The minimum tension in the external parts is zero. Also, in the interior side, as the thickness decreases, the maximum tension decreases too. Meanwhile, as the thickness decreases in the internal parts, the tension increases. However, tension in the vault abutment decreases. Additionally, as the thickness decreases, the thrust force, recession and the minimum imposing pressure on the vault section decrease too (Table 1).

Given that the vault structure resists against the compression forces well (but resists against the tensile forces poorly) it is, therefore, of high importance to find the specific imposing pressure on the arch surface which demonstrates nearly totally compressive behavior. This usually happens when the thrust line is imposed in the middle one-third of the vault. On this basis, the pressure value for the arches with 02, 04 and 06 meters, were calculated as 39.67% , 20.83 % and 46.67% respectively. Therefore, as the thickness of this kind of arch increases, the stability increases too.

Table 1. Analysis of the Panj-o-Haft arch with varying thicknesses in oval method

Thickness (cm)	Minimum imposing pressure	Settlement (KN)	Thrust (KN)	Internal tension (MPa)			External tension (MPa)		
				Abutment	Minimum	Maximum	Abutment	Minimum	Maximum
20	4.3	47.98	19.92	0	0	1.74	1	0	1
40	6.6	100.7	38.41	0.0505	0.0125	1.18	0.0505	0	0.0505
60	49.1	158	59.86	0.0528	0.0156	0.167	0.0528	0	0.0528

### Analysis of the Panj-o-Haft Arch in Compass

#### Method

##### Vault specifications

Span: 4 meters

Thickness: varying

Depth: 5 meters

Density of bricks:  $15 \text{ kN/m}^3$

The results showed that as the thickness decreases, maximum tension in the external and internal part increases. Also, as it decreases from 0.6 to 0.4 meter, the tension of the vault abutment decreases

externally. But, if it decreases to 0.2 meter, the tension increases again. Also, in the interior side of the vault abutment, as the thickness decreases, the tension decreases too. The minimum tension in the internal and external parts is zero. Additionally, as the thickness decreases, the thrust force, recession and the imposing pressure on the vault abutment decrease too (table 2). On this basis, the pressure value for the arches with 02, 04 and 06 meters, was calculated as 32.60 %, 45.20 % and 69.00 % respectively. Therefore, as the thickness of this kind of arch increases, the stability increases too.

**Table 2.** Analysis of the Panj-o-Haft arch with varying thicknesses in compass method

Thickness (cm)	Minimum imposing pressure	Settlement (KN)	Thrust (KN)	Internal tension (MPa)			External tension (MPa)		
				Abutment	Minimum	Maximum	Abutment	Minimum	Maximum
20	10.3	46.31	21.37	0	0	0.802	0.170	0	0.173
40	50.8	97.38	42.16	0.0122	0	0.191	0.0852	0	0.0942
60	81.1	153.2	67.02	0.0128	0	0.0884	0.0894	0.00268	0.0894

### Analysis of the Panj-o-Haft Arch in Numeric

#### Method

##### Vault specifications

Span: 4 meters

Thickness: varying

Depth: 5 meters

Density of bricks:  $15 \text{ kN/m}^3$

Based on the results obtained, the results showed that as the thickness decreases, maximum tension in the external and internal parts and in the vault increases. Also, as the thickness decreases from 0.6 to 0.4 meter, the tension on the vault abutment decreases externally.

But, if it decreases to 0.2 meter, the tension increases again. Also, in the internal side of the vault abutment, as the thickness decreases, the tension decreases too. The minimum tension in the external parts is zero.

Additionally, as the thickness decreases, the thrust force, recession and the imposing pressure on the section vault decrease too (table 3). On this basis, the pressure value for the arches with 02, 04 and 06 meters, was calculated as 28.33 %, 32.33 % and 52.50 % respectively. Therefore, as the thickness of this kind of arch increases, the stability increases too.

**Table 3.** Analysis of the Panj-o-Haft arch with varying thicknesses in numeric method

Thickness (cm)	Minimum imposing pressure	Settlement (KN)	Thrust (KN)	Internal tension (MPa)			External tension (MPa)		
				Abutment	Minimum	Maximum	Abutment	Minimum	Maximum
20	12.1	46.48	21.24	0	0	0.685	0.17	0	0.171
40	43.3	97.71	39.70	0.0489	0	0.199	0.0489	0	0.0777
60	73.8	153.7	61.65	0.0513	0	0.123	0.0513	0	0.0534

### The Analysis of Arches with varying density by Arco Software

In the present study, the stability of Panj-o-Haft arches made of three different materials (adobe, brick and stone) was analyzed. Because these materials are different in their porosity, structure and used mortar, their densities are also different. The approximate densities for adobe, brick and stone were  $12 \text{ kN/m}^3$ ,  $15 \text{ kN/m}^3$  and  $20 \text{ kN/m}^3$  respectively.

##### Vault specifications

Span: 4 meters

Thickness: 0.4 meter

Depth: 5 meters

Density: varying

As the density of materials increases, the maximum tension and tension imposed on the vault abutment – both externally and internally- increase. Also, the thrust force and recession at the vault abutment increase (table 4). The value of thrust line, in all the three modes, is equal to 39.67% aligned with each other. The minimum value of pressure is constant.

### Analysis of the Panj-o-Haft Arch in Oval

#### Method

**Table 4.** Analysis of the Panj-o-Haft Arch with Varying Density in Oval method

Density ( $\text{kN/m}^3$ )	Minimum imposing pressure	Settlement (KN)	Thrust (KN)	Internal tension (MPa)			External tension (MPa)		
				Abutment	Minimum	Maximum	Abutment	Minimum	Maximum
12	6.6	80.53	30.73	0.0404	0.01	0.942	0.0404	0	0.0404
15	6.6	100.7	38.41	0.0505	0.0125	1.18	0.0505	0	0.0505
20	6.6	134.2	51.21	0.0673	0.0167	1.57	0.0673	0	0.0673

## Analysis of the Panj-o-Haft Arch in Compass

### Method

#### Vault specifications

Span: 4 meters  
 Thickness: 0.4 meter  
 Depth: 5 meters  
 Density: varying

As the density of materials increases, the maximum tension and tension imposed on the vault abutment – both externally and internally- increase. Also, the thrust force and recession at the vault abutment increase (table 5). The value of thrust line, in all the three modes, is equal to 51.00% aligned with each other. The minimum value of pressure is constant.

**Table 5.** Analysis of the Panj-o-Haft Arch with Varying Density in Compass method

Density ( $kN/m^3$ )	Minimum imposing pressure	Settlement (KN)	Thrust (KN)	Internal tension (MPa)			External tension (MPa)		
				Abutment	Minimum	Maximum	Abutment	Minimum	Maximum
12	50.8	77.90	33.73	0.00947	0	0.0954	0.0682	0	0.0753
15	50.8	97.38	42.16	0.0122	0	0.119	0.0852	0	0.0942
20	50.8	129.8	56.22	0.0162	0	0.159	0.114	0	0.126

## Analysis of the Panj-o-Haft Arch in Numeric

### Method

#### Vault specifications

Span: 4 meters  
 Thickness: 0.4 meter  
 Depth: 5 meters  
 Density: varying

As the density of materials increases, the maximum tension and tension imposed on the vault abutment – both externally and internally- increase. Also, the thrust force and recession at the vault abutment increase (table 6). The value of thrust line, in all the three modes, is equal to 32.33% aligned with each other. The minimum value of pressure is constant.

**Table 6.** Analysis of the Panj-o-Haft Arch with Varying Density in Numeric method

Density ( $kN/m^3$ )	Minimum imposing pressure	Settlement (KN)	Thrust (KN)	Internal tension (MPa)			External tension (MPa)		
				Abutment	Minimum	Maximum	Abutment	Minimum	Maximum
12	43.3	78.17	31.76	0.0391	0	0.159	0.0391	0	0.0622
15	43.3	97.71	39.70	0.0489	0	0.199	0.0489	0	0.0777
20	43.3	130.3	52.93	0.0652	0	0.265	0.0652	0	0.104

## CONCLUSION

As mentioned before, sustainable architecture tries to achieve a healthy environment based on ecological principles, sensible exploitation of the natural resources and intelligent surveillance of construction process. As a result, the nonrenewable resources will be preserved, the consumption of the renewable energies decreased and the environmental conditions enhanced. Such points are closely related with traditional architecture. For example, the vaults were built in a structurally stable manner with locally available materials and vernacular construction techniques.

The results of the study reveal that in all the three drawing techniques of Panj-o-Haft arch, as the thickness of the vaults increases to 0.6 meter, the value of imposing tension on the vault , both externally and internally, decreases ( despite the increase of thrust force and recession). Therefore, in this arches, as the vault thickness decreases, the pressure force decreases too and, consequently, the more thick an arch is, the better it performs against the imposed pressures (tables 1, 2, 3). In addition, in this arches, with 0.4 meter thickness, as the density increases, the tensions- both externally and internally, thrust, settlement- increase. Therefore, vaults made of adobe are more stable statically (tables 4, 5 and 6). It deserves mentioning, however, that using baked mud as construction material in computerized analysis has been considered irrespective of climatic factors.

In order to compare the determining factors on the stability of vaults based on geometrical parameters, three forms of Panj-o-Haft arches drawn in oval, compass and numeric modes were analyzed. In this comparison the thickness was 0.6 meter and the density was  $12 kN/m^3$  respectively which was the optimum state). In terms of static stability, Panj-o-Haft arches in compass methods outperformed its alternatives.

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