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Diffraction of Water Waves with Arrays of Vertical Cylinders

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ABSTRACT: Based on Linton and Evans (1990) analytical solution, the diffraction of linear water waves by N bottom mounted circular cylinders has been investigated. In order to study the first order interaction between cylinders, the boundary condition on each cylinder surface due to the scattered waves of all other cylinders is taken into account and relations for calculation of forces acting on cylinders are derived. This paper is mainly concerned with vertical cylinders located at vertices of a square. For this configuration the phenomenon of near-trapping mode and the wavelength associated with it is considered and for different wave length the force acting on each cylinder is determined. Also the wave surface elevation around cylinders which can be legs of an offshore platform or piles of a floating island is determined and using MATLAB programming the results are plotted in three dimensional form.

Keywords: Diffraction of water waves; Trapped modes; Hydrodynamic interaction; Array of vertical cylinders.

ORIGINAL ARTICLE

INTRODUCTION

The hydrodynamic interaction of diffracted waves by an array of vertical circular cylinders has received considerable interest in recent years. Such configuration is common in many offshore and onshore structures such as columns of a TLP platform, piles of a floating island, quay piles, connecting bridge columns and columns of very large floating structures (VLFS).

Numerical methods can be used for solution of this problem, however, expensive software or expertise in writing numerical codes is required. An alternative approach is the use of analytical methods. In this regard several various analytical solutions have been developed. Havelock (1940) solved the diffraction problem analytically for a single bottom-mounted circular cylinder in water of infinite depth. McCamy and Fuchs (1954) extended this result to finite depth and this solution has been extended to multiple cylinders by Ohkusu (1974). Detailed reviews of these early works have been given by Mei (1983). Kagemoto and Yue (1986) presented a general solution for diffraction of symmetric cylinders. The problem of scattering of water waves by arrays of fixed vertical circular cylinders is solved exactly by Linton and Evans (1990). Thereafter their solution has been used as the basis of further work by many investigators.

Using a higher order three dimensional spline-Galerkin panel method, Maniar and Newman (1997) found excitation forces acting on circular cylinders that were along an axis and found that near-resonant modes occur between adjacent cylinders when they are at a critical space. In these modes the force acting on each cylinder was very large compared with an isolated cylinder. These modes were associated with the existence of trapped waves in a channel. The existence of such trapped waves had been established by Linton and Evans (1992) and Evans and Porter (1997b). Kagemoto et al. (2002) experimentally analyzed the trapped mode phenomenon for linear arrays of cylinders in a water tank. They found that for incident regular waves at the predicted trapped mode frequencies, the magnification effects were substantially less than predicted by theory. At first, they related this discrepancy to the unsteady flow but further experiments showed that the flow attains a steady state. Then they related the matter to the viscous effects that are neglected in potential flow but they found that the magnitude of drag forces that result from flow separation is small. Finally they considered the viscous dissipative effects within the boundary layers at the cylinder walls, which has been rendered slightly porous to reproduce dissipative effects within the potential flow model, and found very good agreement between theoretical and experimental results. Duclos and Clement (2004) considered the disorders in cylinder spacings and/or radii and found that forces associated with near-trapped modes substantially reduce by disorder of cylinder spacing. They showed that disorder less than 0.5% cylinder spacing for the array is sufficient to reduce the large forces. Instead of using a monochromatic wave, Walker and Taylor (2005) used a model of real sea proposed by Tromans et al. (1991). The diffraction of monochromatic waves by an array of four bottom mounted cylinders using linear and second order theory has been studied by Walker et al. (2008). They presented their results by a numerical diffraction code named DIFFRACT which is written based on the boundary element method.

In this paper first the theoretical solution of Linton and Evans (1990) is reviewed. |Results of this theory related to the vertical cylinders that arranged either in an array arbitrary or located at vertices of a square such as the legs of a TLP platform are presented and discussed. Finally free surface elevation of water around these legs is presented in three dimensional form and near-trapped modes are detected and discussed.

GOVERNING EQUATION

We assume that there are $N(\geq 1)$ vertical circular cylinder(s), each of which extends from the sea bottom in water of uniform depth h. Assuming linear wave theory, the velocity potential for incident waves is:

$$\phi(x,y,z,t) = \mathrm{Re}\left\{\varphi(x,y)f(z)e^{-i\omega t}\right\} \qquad (1)$$

where

$$f(z) = -\frac{igA}{\omega}\frac{\cosh\kappa(h+z)}{\cosh\kappa h} \qquad (2)$$

where xy-plane coincides with the mean free surface and z-axis is vertically upward. A is the wave amplitude, ω is the circular frequency and κ is the wave number. The wave number and frequency are related by the dispersion relation:

$$\omega^2 = g\kappa\tanh\kappa h \qquad (3)$$

The free surface elevation is then given by:

$$\eta(x,y,t) = \mathrm{Re}\left\{A\phi(x,y)e^{-i\omega t}\right\} \qquad (4)$$

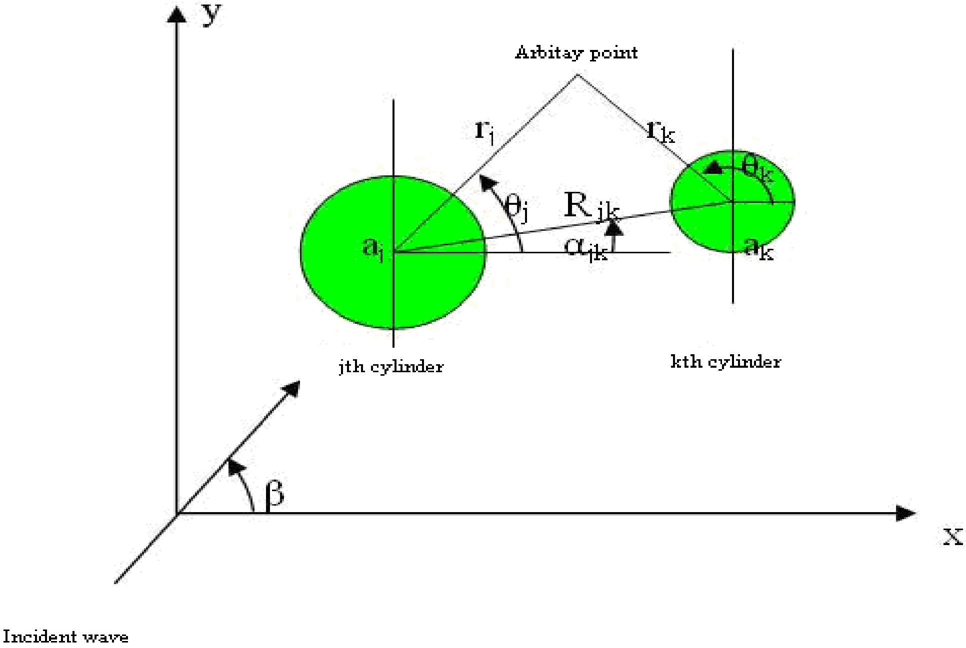

Fig. 1 Plan view of cylinders with various parameters used in relations

For solving this problem we consider N+1 coordinate systems. One of them is used as the reference frame at xy plane and each of the other N systems is located at the center of each cylinder. The origin of each coordinate system is denoted by (x_j, y_j); a_j denotes the radius of jth cylinder, (r, θ) denotes the coordinates of an arbitrary point in reference frame and (r_j, θ_j), $j = 1,\ldots,N$, denotes the coordinates of the same point in the jth coordinate system. Finally, β is the angle of incident wave with positive x-direction. The velocity potential of an incident plane wave propagating at an angle β to the x-axis can be expressed in the following form:

$$\varphi_I = e^{i\kappa(x\cos\beta + y\sin\beta)} = e^{i\kappa r\cos(\theta-\beta)} = I_j e^{i\kappa r_j\cos(\theta_j-\beta)} \qquad (5)$$

Where, I_j is a phase factor associated with jth cylinder and can be written as:

$$I_j = e^{i\kappa(x_j\cos\beta + y_j\sin\beta)} \qquad (6)$$

The total potential function by considering the linear interaction of diffracted waves from each cylinder can be written as:

$$\varphi = \varphi_I + \sum_{j=1}^{N}\varphi_s^j \qquad (7)$$

where $\sum_{j=1}^{N}\varphi_s^j$ is the potential function of diffracted waves. The incident velocity potential, φ_I, is now expressed as a sum of Bessel functions (Gradshteyn and Ryzhik, 2000):

$$e^{i\kappa r\cos\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(\kappa r)e^{in\theta} \qquad (8)$$

This leads to:

$$\varphi_I = I_j\sum_{n=-\infty}^{\infty} i^n J_n(\kappa r_j)e^{in(\theta_j-\beta)} \qquad (9)$$

and since $i = e^{i\frac{\pi}{2}}$ then $i^n = e^{in\frac{\pi}{2}}$ and we have

$$\varphi_I = I_j\sum_{n=-\infty}^{\infty} J_n(\kappa r_j)e^{in(\frac{\pi}{2}+\theta_j-\beta)} \qquad (10)$$

The general form of the diffracted potential associated with jth cylinder can also be expressed as:

$$\varphi_s^j = \sum_{n=-\infty}^{\infty} A_n^j Z_n^j H_n(\kappa r_j)e^{in\theta_j} \qquad (11)$$

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For the same set of complex numbers A_n^j. Where

$$Z_n^j = Z_{-n}^j = \frac{J_n'(\kappa a_j)}{H_n'(\kappa a_j)}, \qquad H_n(\kappa r_j) = J_n(\kappa r_j) + iY_n(\kappa r_j),$$

J_n and Y_n are the Bessel functions and a_j is the radius of jth cylinder. In the present study a_j is considered to be identical for all cylinders. By using Graf's addition theorem for Bessel functions and by imposing the boundary condition on cylinder surfaces, Linton and Evans (1990) derived an infinite system of equations with infinite A_n^j unknowns as follows:

$$A_m^k + \sum_{\substack{j=1 \\ j\neq k}}^{N} \sum_{n=-\infty}^{\infty} A_n^j Z_n^j H_{n-m}(\kappa R_{jk}) e^{i(n-m)\alpha_{jk}} = -I_k e^{im(\frac{\pi}{2}-\beta)}$$

$$k = 1,\dots,N, \qquad -\infty < m < \infty \tag{12}$$

where α_{jk} is the angle that connector line from center of jth cylinder to kth cylinder makes with the positive x-direction and R_{jk} is the distance between the center of cylinders jth and kth, as shown in figure 1. Since Graf's addition theorem is valid if $r_k < R_{jk}$ for all j, therefore, the derived potential function is valid as far as this condition is satisfied for each cylinder. By using (12), the potential function around each arbitrary cylinder such as kth cylinder can be written as follows (Linton & Evans 1990):

$$\varphi(r_k,\theta_k) = \sum_{n=-\infty}^{\infty} A_n^k (Z_n^k H_n(\kappa r_k) - J_n(\kappa r_k)) e^{in\theta_k}$$

$$\text{if} \quad r_k < R_{jk} \qquad \forall j. \tag{13}$$

Now in order to evaluate the constant unknowns, A_j^n, a finite system of equations rather than an infinite system of equations can be considered. This can be done by selecting a limited value for m boundaries which returns N(2M+1) equations and N(2M+1) unknowns:

$$A_m^k + \sum_{\substack{j=1 \\ j\neq k}}^{N} \sum_{n=-M}^{M} A_n^j Z_n^j H_{n-m}(\kappa R_{jk}) e^{i(n-m)\alpha_{jk}} = -I_k e^{im(\frac{\pi}{2}-\beta)}$$

$$k = 1,\dots,N \quad , \quad m = -M,\dots,M. \tag{14}$$

The value of M can be increased until the desired accuracy is achieved. It is obvious that by increasing M, the expense and duration of computing will be increased. We shall discuss about the efficient value of M in this article. Now the first-order force acting on the jth cylinder can be obtained by integrating the pressure over the surface of the cylinder, that is,

$$\left|F^j\right| = \frac{1}{2}|F| \left| (A_{-1}^j \begin{Bmatrix} - \\ + \end{Bmatrix} A_1^j) \right| \tag{15}$$

where

$$F = \frac{4\rho g A \tanh \kappa h}{\kappa^2 H_1'(\kappa a)} \tag{16}$$

In which, F^j is the force acting on jth cylinder, F is the force acting on a similar isolated cylinder in the same incident wave train and ρ is the density of water. The minus and plus signs in the bracket corresponds to the x- and y- components, respectively. Equation (15) can be rewritten as:

$$\left|\frac{F^j}{F}\right| = \frac{1}{2} \left| (A_{-1}^j \begin{Bmatrix} - \\ + \end{Bmatrix} A_1^j) \right| \tag{17}$$

This equation can be used for comparison of the forces acting on a specific cylinder when it is either in an array or is isolated and is under the same wave conditions in both cases. By using (4) the free surface elevation, can be obtained from the following equation:

$$|\eta(x,y,t)| = |A\varphi(x,y)| \tag{18}$$

or

$$|\eta| / A = |\varphi(x,y)| \tag{19}$$

where $\varphi(x,y)$ could be derived from (13) by considering the relevant boundary condition for each cylinder.

CALCULATION OF FORCES ACTING ON CYLINDERS LOCATED IN ROWS

By using the results of previous section, the truncated system of equations is solved for various value of M and the A_n^j unknowns are computed. Then by using equation (16) the forces acting on cylinders are derived. This is down by writing a program in MATLAB. The force acting on the middle cylinder for arrays of 3 and 9 cylinders compared with an isolated cylinder at the same wave feature for a certain frequency interval is shown in figures 2 and 3. For both cases $\beta = 0$, $a/d = 1/4$ and $M = 4$. Cylinders are equally spaced along a line and the distance between adjacent cylinders is 2d. All cylinders has the same radius which is denoted by a. The obtained results coincide with those obtained by Maniar and Newman (1997) using a high order three dimensional spline-Galerkin method, and with those of Walker and Taylor (2005). Figures 2 and 3 show the significance of the parameter κd, which relates the spacing between adjacent cylinders to the wavelength, on the force magnitude. As seen in figures 2 and 3 the critical modes with narrow peaks occur when κd is slightly less than $\pi/2$ multiples. Moreover as Walker and Taylor (2005) mentioned the magnitude of peaks increases as the number of cylinders in a row increase; for a 9-cylinder array the highest peak is approximately 3 times the force on a single isolated cylinder. For a 3-cylinder array the multiplicative factor is approximately 1.5.

As another result these figures indicate that as the number of cylinders in an array increases, the bandwidth of critical modes for the middle cylinder reduces rapidly (i.e., peaks become narrower). The frequencies at which these magnification effects occur are referred to as near-trapped mode frequencies. Trapped modes are of two types, namely Numann trapped modes and Dirichlet trapped modes. The Numann trapped modes, satisfy Neumann conditions on all solid boundaries and it is shown that a Dirichlet condition exists on the center plane for $0 < a/d < 1$. Detailed description of trapped modes is given by Linton and Evans (1992) and Evans and Porter (1997b).

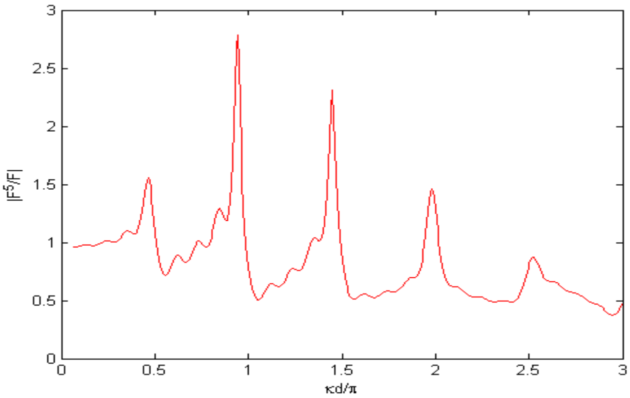


Figure 2. Force on middle cylinder in an array of nine cylinders divided by the force on a single isolated cylinder

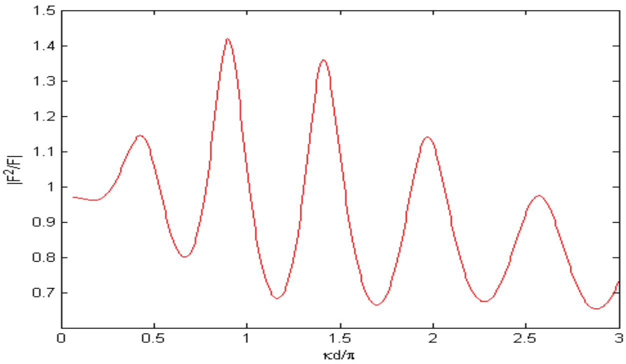
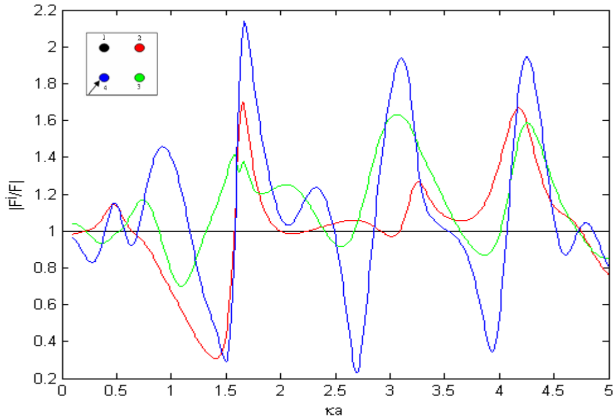


Figure 3. Force on middle cylinder in an array of three cylinders divided by the force on a single isolated cylinder.

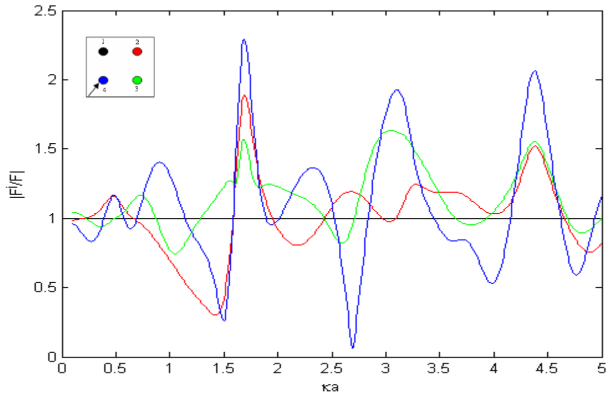
An interesting example is the case of four cylinders arranged at vertices of a square. For this configuration the forces acting on cylinders are computed for values of $a/h = 0.5$, $R/h = 2$ and $\beta = \pi/4$. Each cylinder is located at the vertices of a square of side R at coordinates (-h, h), (h, h), (h, -h), (-h, -h) which named 1, 2, 3 and 4, respectively. The results are obtained for various κa values. Also the free surface elevation around legs and the space between them in critical modes are evaluated in this paper.

The ratio of force acting on each cylinder to an isolated cylinder in the direction of incident wave for various κa values and for M values chosen from 1 to 6 is shown in fig 4. As can be seen the force acting on first and third cylinders is identical, in the figure the related curves are coincided. It should be mentioned that the results published by Linton and Evans (1990) for this arrangement of cylinders are in error as illustrated by Linton and McIver (2001).

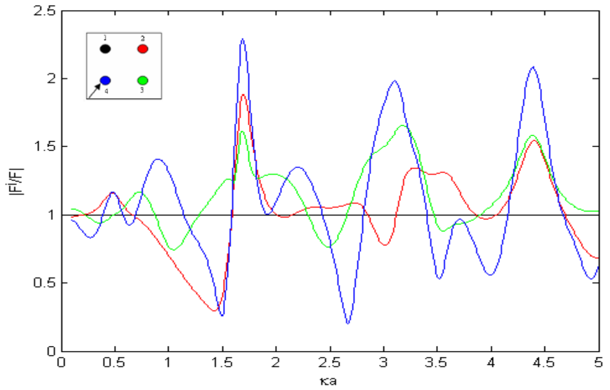
Figures (a) to (f) show that the critical values of force acted on cylinders occur when κa have slightly skew around multiples of 1.5. and the highest peaks occur for κa around the first multiple (for $M \geq 4$). It is shown that the maximum force is acting on the forth cylinder and the minimum force is acting on cylinders 1 and 3. The bandwidth corresponding to the first critical κa is less than that of the other critical states, therefore, we can say that the wave-number in this state is related to near-trapping mode.



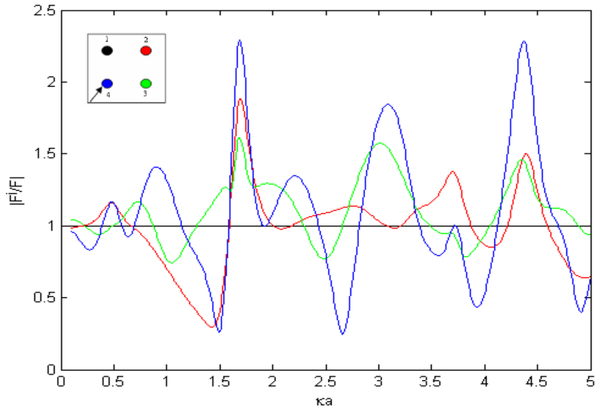
(Figure 4, a)



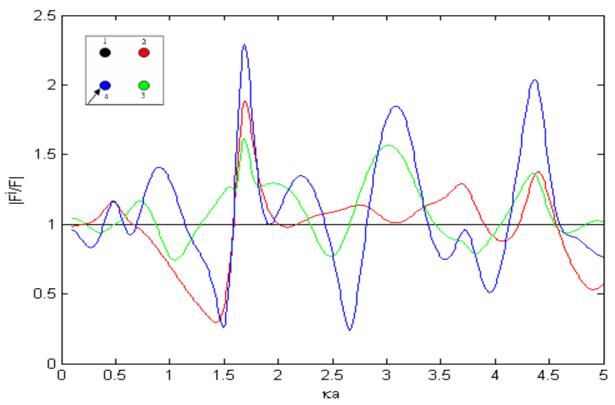
(Figure 4, b)



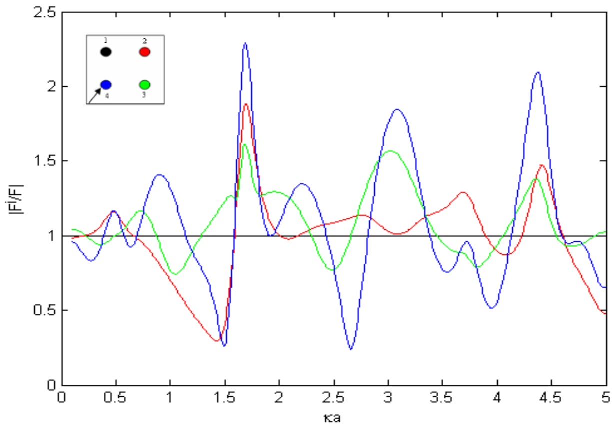
(Figure 4, c)



(Figure 4, d)



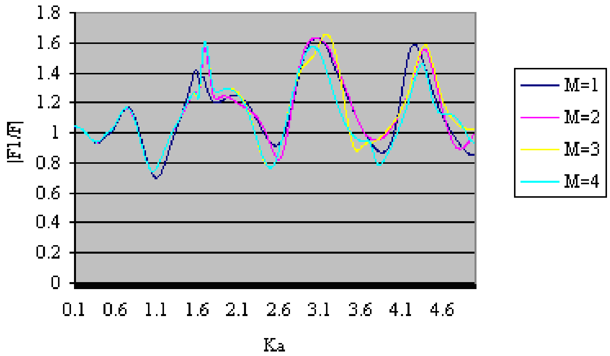
(Figure 4, e)



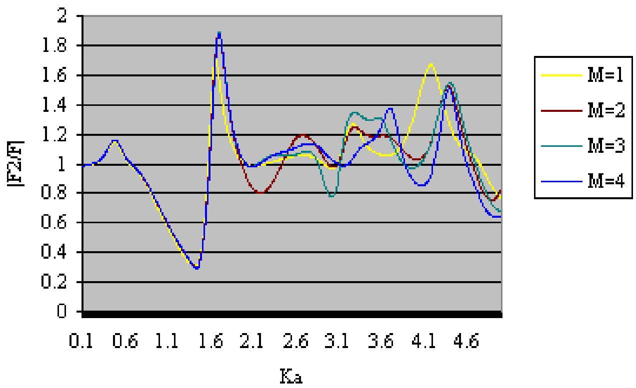
(Figure 4, f)

Figure 4. Non-dimensional first order force acting on cylinders located at vertices of a square in the direction of wave advance according to the coordinates in text, for: (a) M=1, (b) M=2, (c) M=3, (d) M=4, (e) M=5, (f) M=6.

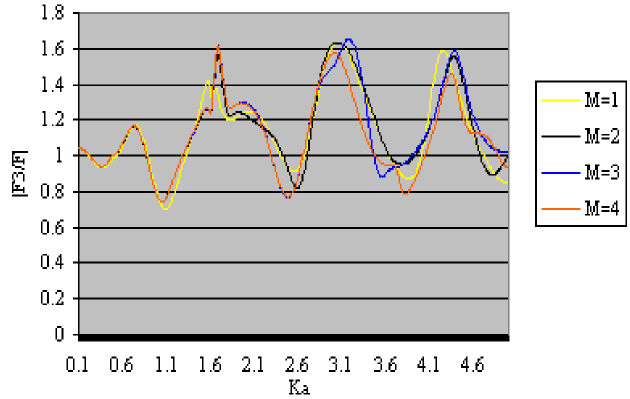
Also these figures indicate that for $M < 4$ the results are not sufficiently accurate and for $M \geq 4$ the difference between results is negligible. To assess this, we transferred the results obtained by the written program in MATLAB to an Excel spread sheet. The force acting on each cylinder obtained by varying the M value is shown in fig. 5. In order to make the figure more readable, the results for each cylinder are plotted separately. According to these figures we can say that by increasing the wave number (note that the radius is assumed constant), the deviance in results by varying of M value increases. Also before the first near-trapping mode, the deviance of data is negligible regardless of the value of M and after the first critical mode, the deviance of results is started.



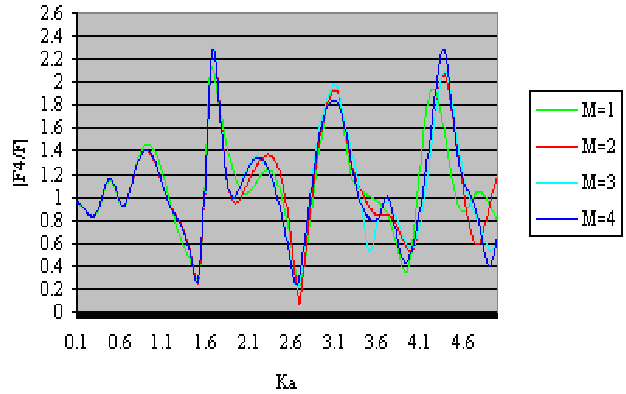
(Figure 5, a)



(Figure 5, b)



(Figure 5, c)



(Figure 5, d)

Figure 5. Non-dimensional first order force acting on cylinders located at the vertices of a square in the direction of wave advance according to coordinates in text, for various M values: a= cylinder 1, b= cylinder2, c= cylinder 3, d= cylinder 4.

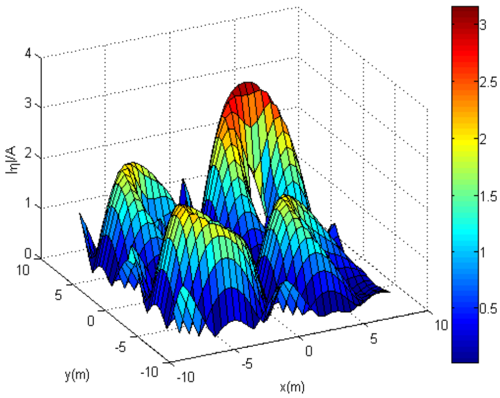
In table 1 the ratio of the maximum force acting on each cylinder to the force acting on an identical isolated cylinder is shown for various values of M. By looking at the table we find that the κa value corresponding to the near-trapping mode is 1.69. In this case the force acting on the forth cylinder is 2.5 times the force acting on a similar isolated cylinder. Evans and Porter (1997) and walker et al. (2008) showed that for the same case but with $h = 3a$, this value is approximately equal to 1.66.

Evaluating the Free Surface Elevation around Cylinders

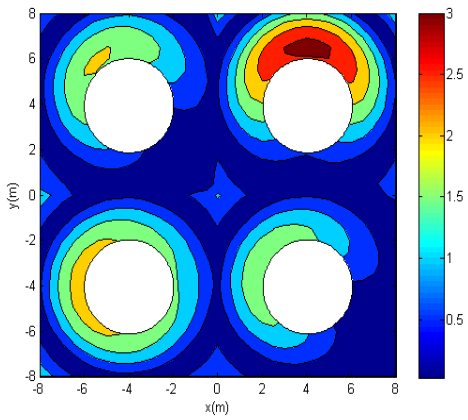
In this section we shall consider the free surface elevation of linear water waves incident upon cylinders that are located at the vertices of a square. By solving the system of equations for $M=4$, $\beta=\pi/4$ and κa values corresponding to peak force values, i.e., values between 1.65 and 1.69, the unknown coefficients are derived. Then for $R=4a$ and $h=2a$ by using equation (13) and considering the Graf's addition theorem, the magnitude of $\varphi(x,y)$ and the three dimensional surface elevation around each cylinder within its validated zone are calculated. The results for $\kappa a=1.69$ and $\kappa a=1.66$ are shown in figures 6 and 7. According to these figures, free surface elevation of water waves on or close to the cylinders surface are increased and at the zone between cylinders is relatively small. For $\kappa a=1.66$ we have the largest free surface elevation that occurs around the cylinder 2. Its magnitude is 3.5 times the incident wave amplitude whereas in $\kappa a=1.69$ it is 3 times the incident wave amplitude hence for a near-trapped mode, the maximum force and free surface magnification do not necessary occur at exactly the same frequency.

Table 1. Maximum of non-dimensional first order force acting on cylinders located at the vertices of a square in the direction of wave advance according to coordinates in text, for various M values and corresponding κa values.

Cylinder No. ↓	M value →	1	2	3	4	5	6
1	$\lvert F_{max}/F \rvert$	1.632707	1.632763	1.656199	1.611353	1.611615	1.611618
	κa	3.06	3.05	3.18	1.69	1.69	1.69
2	$\lvert F_{max}/F \rvert$	1.698939	1.889397	1.883363	1.879945	1.880347	1.880353
	κa	1.65	1.69	1.7	1.69	1.69	1.69
3	$\lvert F_{max}/F \rvert$	1.632707	1.632763	1.656199	1.611353	1.611615	1.611618
	κa	3.06	3.05	3.18	1.69	1.69	1.69
4	$\lvert F_{max}/F \rvert$	2.138904	2.293751	2.294089	2.292347	2.292631	2.292639
	κa	1.67	1.69	1.69	1.69	1.69	1.69

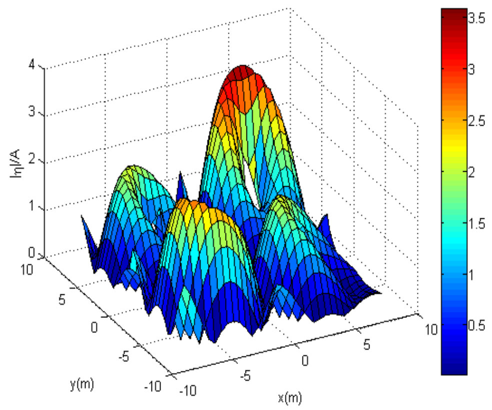


(a) Surface plot

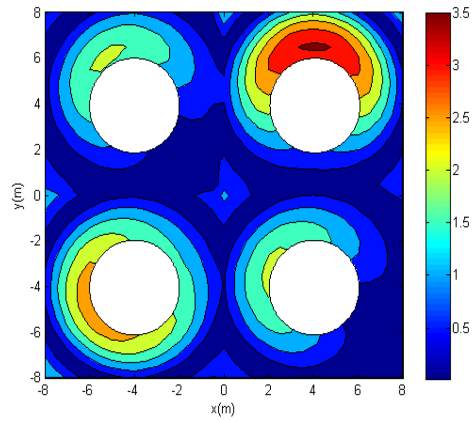


(b) Contour plot

Fig. 6 Free surface elevation of water waves around a four-cylinder array (R=4a, h=2a, κa=1.69).



(a) Surface plot



(b) Contour plot

Fig. 6 Free surface elevation of water waves around a four-cylinder array (R=4a, h=2a, κa=1.66).

CONCLUSIONS

By using the interaction theory of Linton and Evans (1990), the forces acting on the cylinders that are located in a linear array, are calculated. When cylinders are located at the vertices of a square, the force and free surface elevation for critical κa values were analyzed.

It was shown that by increasing the number of cylinders in linear arrays, the force acted on the middle cylinder increases, also the bandwidth of trapped modes decreases and the required number of equations to achieve the sufficient accuracy increases.

By studying the results for cylinders located at vertices of a square it was found that before the occurrence of the first near-trapped mode there is a considerable coincidence in results for various number of equations but after the occurrence of the first near-trapping mode the deviance of data especially deviance between the results when the number of equations is less than 9 ($M < 4$) and from equations more than these states increased.

The results obtained from MATLAB software can show the patterns about maximum loads acted on cylinders that used as supports of various marine structures and the free surface elevation around and between them, that by using of these results we can find the best geometry situation for cylinders to have optimum serviceability.

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