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Vertical Sluice Gate Discharge Coefficient

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ORIGINAL ARTICLE

ABSTRACT: Sluice gates are widely used for flow control and discharge measurement in irrigation and drainage channels. Their discharge coefficient depends on geometric and hydraulic parameters. Errors are inevitable when their values are abstracted from empirical curves for a range of reasons including the resolution of the graphs, judgments in reading the values, the need for their digital values at the computational stages. This study develops two equations, linear and nonlinear, to determine discharge coefficient by using dimensional analysis and linear and nonlinear regression analysis, for both free and submerged flow conditions. A total of 5200 data point was generated, which involved different effective hydraulic parameters. The study also included the results of the past studies carried out by different investigators concerning sluice gate discharge coefficient determination for comparison purposes. The performance of the nonlinear equation improves in comparison with the linear equation. All numerical computations were carried out by Wolfram Mathematica v.6 software.

Keywords: Discharge coefficient, free flow, multiple regressing, sluice gate, submerged flow.

1. INTRODUCTION

Discharge can conveniently be measured by hydraulic structures for controlling discharge and water depth, as they create a one-to-one relationship between depth and discharge. Their applications include irrigation and drainage canals and overflow spillways. Notably, other discharge measuring device are costly, e.g. Laser Doppler Anemometer. Attention to the understanding of the performance of weir-type flow control structure, e.g. over-flow and sharp-crested weirs is relatively better than that of standard gates. The basic knowledge of the hydraulic performances of gated structures is even poorer than of the vibration of these gates, e.g. see [1].

Open channel flow software modelling has become standard design tools for irrigation canals. Over the years, research was focused on developing numerical schemes for the solution of the shallow-water equations but sometimes this was at the expense of overlooking the significance of other components, e.g. the performance of sluice gates, e.g. see [2]. The mode of flow associated with gated structures is often complex under real-time conditions but their hydraulics fall into two regimes of: (i) modular flows when discharge is independent of the tailwater depth; and (ii) submerged flows when there is dependency; see Fig. 1.

This distinction is well established as the submergence reduces the discharge through the gates and this is reflected on the values of discharge coefficient. Figure 1 shows that that submerged flow occurs when the tailwater depth is greater than the downstream depth of the hydraulic jump, y_3.

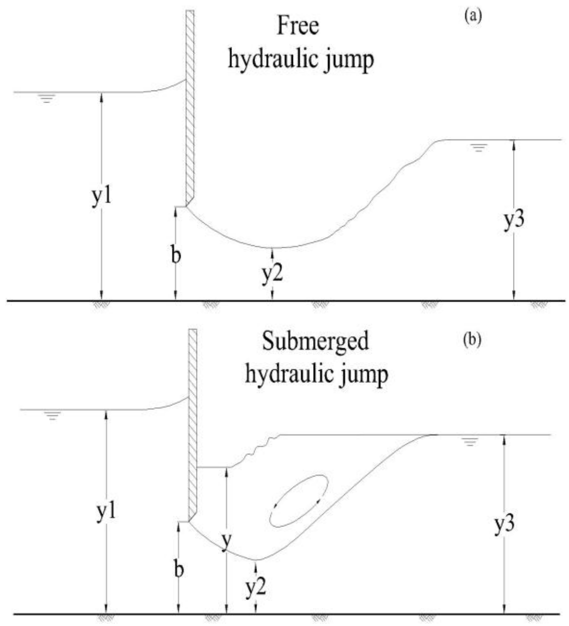

Figure 1 Scheme of a gate operating under free (a) and submerged (b) flow condition.

A description of flow equations has been approached by theoretical and empirical formulas and by graphical approaches. Swamee (1992) presents two formulas to distinguish modular and submerged flow conditions based on Henry's (1950) curve. Yen et al. (2001) presents a theoretical formula and some experimental graphs to determine maximum allowable

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tailwater depth for free flow (or minimum allowable tailwater depth for submerged flow). They define the condition separating free flow and submerged flow in terms of flow contractions at the gate and derive equations for discharge coefficient in terms of dimensionless discharge, submerged water depth, maximum allowable gate opening. They also compare their results with other investigators approach and report good fitness.

In this study we've used Swamee's (1992) approach to determine whether the jump will be free or submerged.

Free flow $$y_1 \geq 0.81 y_3 \left(\frac{y_3}{b}\right)^{0.72} \quad (1)$$

Submerged flow $$y_3 < y_1 < 0.81 y_3 \left(\frac{y_3}{b}\right)^{0.72} \quad (2)$$

where y_1 = upstream depth, y_3 = tailwater depth and b = gate opening.

Discharge Formulation

Using the Bernoulli's and continuity equations in sluice gate hydraulic jump flow, it is possible to derive the following widely known expression to calculate the gate discharge for a rectangular cross section:

$$q = C_d b \sqrt{2gy_1}$$

where q = discharge per unit width of channel, g = gravity acceleration, b = gate opening, y_1 = upstream depth and C_d = discharge coefficient. C_d depends on different parameters such as upstream and tailwater depths, gate opening, contraction coefficient of gate and the flow condition.

Eq. (3) is applicable for both hydraulic conditions.

Henry (1950) used Eq. (3) and evaluated C_d experimentally. The outcome of this study is the well-known Henry's curve.

Henderson (1966) derived two equations to compute C_d for each flow condition.

Free flow $$C_d = \frac{C_c}{\sqrt{1+\eta}} \quad (4)$$

Submerged flow

$$C_d = C_c \frac{\left(\xi - \sqrt{\xi^2 - (1/\eta^2 - 1)^2 (1 - 1/\lambda^2)}\right)^{1/2}}{1/\eta - \eta} \quad (5)$$

Where

$$\eta = C_c b / y_1 \ , \quad \lambda = y_1 / y_3 \ , \quad \xi = ((1/\eta) - 1)^2 + 2(\lambda - 1)$$

and C_c = contraction coefficient.

The contraction coefficient is defined as the ratio of the water depth at vena contracta, y_2 to gate opening ($C_c = y_2 / b$).

For sharp-edge vertical sluice gate C_c varies between 0.598 and 0.611 based on theoretical reasons [5]. Since the contraction coefficient depends on gate opening, shape of the gate lip, upstream water depth, gate type, and so forth, it is very difficult to know its real value for all operating conditions in practice [7]. For practical purposes, selecting $C_c = 0.61$ have an accurate results and many researchers have used this value [6].

Another study of sluice gate discharge calculation was performed in Rajaratnam and Subramanya (1967). They expressed the discharge through a sluice gate as

$$q = C_d b \sqrt{2g(y_1 - C_c b)} \quad (6)$$

$$q = C_d b \sqrt{2g(y_1 - y)} \quad (7)$$

A value of 0.61 was used for C_c and the analysis of experimental data indicated that C_d was uniquely related to b/y_1 for both flow conditions. For $b/y_1 < 0.3$ this relationship was almost linear with Eq. (8).

$$C_d = 0.0297\, b/y_1 + 0.589 \quad (8)$$

As can be noted, Eq. (7) makes use of gate submergence depth, y (see Fig. 1). Because it is very difficult to accurately measure its value (this zone has standing recirculation flows), it must be predicted. After certain simplifications they obtained

$$y = bC_d\left[2\left(1 - \frac{bC_d}{y_3}\right) + \sqrt{4\left(1 - \frac{bC_d}{y_3}\right)^2 + \left(\frac{y_3}{bC_d}\right)^2 - 4\left(\frac{y_1}{bC_d} - \frac{y_1}{y_3}\right)}\right] \quad (9)$$

Swamee (1992) obtained discharge coefficient equations for free and submerged flow, by performing nonlinear regression on Henry's (1950) curve.

Free flow $$C_d = 0.611\left(\frac{y_1 - b}{y_1 + 15b}\right)^{0.072} \ (3) \quad (10)$$

Submerged flow

$$C_d = 0.611\left(\frac{y_1 - b}{y_1 + 15b}\right)^{0.072}\left(0.32\left(\frac{0.81 y_3 \left(\frac{y_3}{b}\right)^{0.72} - y_1}{y_1 - y_3}\right)^{0.7} + 1\right)^{-1} \quad (11)$$

MATERIALS AND METHODS

First we consider free flow condition. Relation among hydraulic parameters can be determined by applying the Bernoulli's equation between sections 1 and 2, and specific force equation between sections 2 and 3 in Fig. 1-a. Considering the channel bottom as the datum and neglecting the energy losses at the gate, Bernoulli's and specific force equations yield Eq. (12)and (13).

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} \quad (12)$$

$$\frac{y_2^2}{2} + \frac{q^2}{gy_2} = \frac{y_3^2}{2} + \frac{q^2}{gy_3} \quad (13)$$

Combination of Eq. (12) and (13) results Eq. (14).

$$f(y_1, y_3, q, y_2) = \left(y_1 + \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right)\right)^2 + \frac{2q^2}{g}\left(\frac{1}{y_2} - \frac{1}{y_3}\right) - y_3^2 = 0 \quad (14)$$

By selecting different (but hydraulically feasible) values for y_1, y_3 and q, we can solve Eq. (14) with respect to y_2. Then we can calculate gate opening b ($b = y_2/0.61$). After that using Eq. (1) and (2) we could specify flow condition. Now we can calculate discharge coefficient by presented formulas for both free and submerged flow.

All of the computations were carried out by Mathematica v.6 software. The input data (y_1, y_3 and q) generation process is programmed to generate random real numbers between following defined limits (It's notable that SI system of units was used in all over this paper):

$$0.1 \le y_1 \le 5\ ,\quad 0.1 \le y_3 < y_1\ ,\quad 0.005 \le q \le 2$$

These values are which practically occurs in irrigation and drainage channels. It's notable that the tailwater depth always must be less than the upstream depth otherwise the flow direction will be reverse and if these two depths be equal, then there is no flow and discharge coefficient will be zero.

At any point three input data are generating at the same time and are substituting in Eq. (14). Each data point yields four values for y_2 as the roots of Eq. (14) in which two of them always are with minus sign and are not practically acceptable values. The third and fourth roots are complex numbers for some data points, so these data points should be eliminated from data series. Use of the third real roots at calculation process yields negative values for y in submerged condition, so this root is not acceptable too, and only the natural values of fourth root at any data point will be used in future steps.

Now by specifying the values of y_2 and contraction coefficient, the gate opening could be calculated. In some data points, y_2 and then the gate opening is a very small value which is certainly not feasible. If these data points be used in following processes (dimensional analysis and dimensionless numbers generation), unusual and digressive values will obtain which affects regressing result and decreases the fitness of fitted relations. So data points with a gate opening less than 0.05 m are eliminated.

Initially generated data sets of y_1, y_3 and q were selected about 10,000 points ,in order to beset all of the possible situations, but after ignoring unacceptable data points, finally 5200 data set were remained, and it is about 45 % of initial points. 650 data points of the remained set are about the free flow and 4550 data points are about the submerged flow condition.

A part of generated data points and calculated parameters are presented in Table 1.

At submerged flow the water depth at the immediately downstream of the gate (section 2 in Fig. 1-b) is y. So y must be used as the piezometric head at the right side of the Bernoulli's equation, but at velocity head term, y_2 is used as flow section depth because water flows only from vena contracta and there is just a stationary circulating flow at the upper part of the vena contracta. So Bernoulli's equation alters to Eq. (15).

$$y_1 + \frac{q^2}{2gy_1^{\,2}} = y + \frac{q^2}{2gy_2^{\,2}} \tag{15}$$

After finding y_2 values, by substituting y_1, y_2 and q at Eq. (15), y values will be obtained which are presented in Table 1, too. For free flow y equals to y_2 (see Table 1). For Rajaratnam and Subramanya (1967) method we couldn't use Eq. (8) to determine C_d when b/y_1 is greater than 0.3, so no calculation were done to compute C_d value for this situation. This condition is specified by "-" in Table 1. yR column in Table 1 corresponds the solution of Eq. (9). In addition, C_dH, C_dR and C_dS columns in Table 1 are discharge coefficients obtained by Henderson (1960), Rajaratnam and Subramanya (1967) and Swamee (1992), respectively.

Dimensional Analysis

Effective hydraulic parameters are as follows for flow through sluice gate: $C_d, q, g, b, y_1, y, y_3$. By rearranging theses parameters as dimensionless parameters, we have

$$C_d = F\left(\frac{gb^3}{q^2}, \frac{y_1}{b}, \frac{y}{b}, \frac{y_3}{b}\right) \tag{16}$$

As can be seen, in addition to y_1/b and y_3/b which were used in Henry (1950) and Swamee (1992), there are some other parameters involved in this phenomenon. Dimensionless parameter q^2/gb^3 is Froude Number regarding to gate opening. So Eq. (16) can be alters to Eq. (17).

$$C_d = F\left(\frac{1}{F_r^{\,2}}, \frac{y_1}{b}, \frac{y}{b}, \frac{y_3}{b}\right)$$

All independent parameters in Eq. (17) are contributing in submerged flow condition and only two of them ($1/Fr^2$ and y_1/b) are contributing in the free flow condition.

We can specify F by performing experimental studies for obtaining data or by generating these required data from analysing governing equations and by the use of computers. As mentioned previously, second method was used in this study.

At this step multiple regression techniques were used to determine F regarding to generated and then refined data series. For any flow condition a linear and nonlinear functions are defined and regressing procedure is performed base on these functions. Linear and nonlinear regressing pattern is as following, respectively

$$(C_d)_i = \alpha_0 + \alpha_1\left(\frac{y_1}{b}\right)_i + \alpha_2\left(\frac{y_3}{b}\right)_i + \alpha_3\left(\frac{y}{b}\right)_i + \alpha_4\left(\frac{1}{Fr^2}\right)_i \quad , \quad i = 1,2,\ldots,n \tag{18}$$

$$(C_d)_i = \beta_0\left(\frac{y_1}{b}\right)_i^{\beta_1}\left(\frac{y_3}{b}\right)_i^{\beta_2}\left(\frac{y}{b}\right)_i^{\beta_3}\left(\frac{1}{Fr^2}\right)_i^{\beta_4} \quad , \quad i = 1,2,\ldots,n \tag{19}$$

where n is number of data points and α and β are unknown parameters that regressing target is to find them. Ordinary least square (OLS) method was used to determine these parameters.

Eq. (18) and (19) are arranged for submerged condition and as mentioned previously y/b and y_3/b didn't used in free flow condition.

RESULTS

In this study by the use of Eq. (14) and (15), a code was written in Mathematica v.6 to coincide solve of Bernoulli's an specific force equations. This code can omit negative and complex roots of Eq. (14) and (15), also it is designed to generate applicable values for engineering purposes. After generation of geometrical and hydraulic properties of flow, the flow condition whether free or submerged was determined. After that, the discharge coefficient computed by Henderson (1966), Rajaratnam and Subramanya (1967), and Swamee (1992) approaches and finally a sample of 5200 data points presented in Table 1.

Then using the computed discharge coefficients, discharge passes through the gate for each data points computed by Eq. (3) or (6) and (7). Table 2 presents a part of computed discharge values by three methods and initially generated values.

The error introduced by each method is determined by the mean absolute percentage error (MAPE). MAPE is defined as follows

$$MAPE = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{q_i - \hat{q}_i}{q_i}\right| \quad (20)$$

where q_i is initially generated discharge and $\hat{q}_i$ is -calculated values of discharge by different researchers formulas and n is total number of data points.

The MAPE for Henderson (1966), Rajaratnam and Subramanya (1967), and Swamee (1992) methods is 6.73%, 4.43% and 23.6% respectively which have a good accordance with Sepúlveda et. al (2009) results.

According to the definition of MAPE criteria, the approach of Rajaratnam and Subramanya (1967) has more accuracy to state flow through sluice gates, comparison with two other methods. It is notable that the discharge coefficient computed by this method is approximately constant value of 0.59, nevertheless this method has a high accuracy for computation of discharge rate. The only restriction of this method is the $b/y_1 < 0.3$ constrain which is satisfied for 4151 of 5200 data points.

Table 1. A part of generated and calculated parameters by Mathematica code.

No.	y_1	y_3	q	y_2	y	yR	b	Fr	Condition	C_dH	C_dR	C_dS
1	2.03978	1.29503	1.22186	0.24896	0.83040	0.8681	0.40746	1.49985	Sub	0.4740	0.5949	0.4546
2	3.62529	3.51484	1.63123	0.84236	3.44448	-	1.37867	0.32172	Sub	0.1402	-	0.1515
3	0.37555	0.16531	1.37889	0.22713	0.22713	-	0.37173	1.94240	Free	0.4823	-	0.3598
4	3.40270	2.82733	0.53478	0.15066	2.76177	2.7641	0.24658	1.39446	Sub	0.2654	0.5911	0.1726
5	2.22315	0.16703	1.25910	0.23386	0.23386	0.7220	0.38275	1.69761	Free	0.5811	0.5941	0.5498
6	2.85622	2.61771	0.36297	0.15767	2.58695	2.5880	0.25806	0.88399	Sub	0.1878	0.5916	0.1172
7	2.59109	1.79507	1.25271	0.26743	1.48465	1.5009	0.43769	1.38121	Sub	0.4014	0.5940	0.3716
8	3.61884	3.28892	0.11077	0.04296	3.28017	3.2804	0.07032	1.89635	Sub	0.1869	0.5895	0.0598
9	1.67327	1.34993	1.28254	0.36472	1.07297	-	0.59693	0.88786	Sub	0.3749	-	0.3986
10	2.11579	1.86706	1.99555	0.60435	1.60542	-	0.98912	0.64767	Sub	0.3131	-	0.3745
11	3.14932	2.41600	0.16433	0.04256	2.38956	2.3905	0.06965	2.85400	Sub	0.3001	0.5896	0.1339
12	2.47770	1.45313	0.51334	0.10584	1.28101	1.2878	0.17323	2.27315	Sub	0.4250	0.5910	0.3449
13	3.77669	0.15417	1.71330	0.23991	0.23991	1.1289	0.39265	2.22321	Free	0.5924	0.5920	0.5665
14	1.60619	0.92361	0.80825	0.16946	0.45973	0.5085	0.27736	1.76661	Sub	0.5191	0.5941	0.4898
15	1.40228	0.21594	1.39896	0.49191	0.49191	-	0.80509	0.61829	Free	0.52571	-	0.4881
16	2.31773	1.16745	0.68530	0.12665	0.83006	0.8468	0.20729	2.31826	Sub	0.4902	0.5916	0.4449
17	1.43791	0.77458	0.16806	0.04385	0.69	0.6932	0.07177	2.79062	Sub	0.4408	0.5904	0.3434
18	0.72363	0.6313	0.18905	0.11472	0.58869	0.5914	0.18775	0.74192	Sub	0.2672	0.5967	0.2608
19	3.16135	1.19207	0.72567	0.10423	0.69360	0.7217	0.17059	3.28820	Sub	0.5401	0.5906	0.5012
20	3.12923	2.95729	1.64828	0.68813	2.85095	-	1.12624	0.44029	Sub	0.1867	-	0.2012
21	3.53052	3.26738	0.87167	0.34327	3.20498	3.2076	0.56182	0.66087	Sub	0.1864	0.5937	0.1449
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5193	1.43763	1.04689	1.03028	0.26605	0.6995	-	0.43544	1.14479	Sub	0.4455	-	0.4501
5194	3.57745	2.93680	1.80137	0.43233	2.70552	2.7174	0.70758	0.96628	Sub	0.3038	0.5948	0.2786
5195	4.99268	4.81000	1.48372	0.67386	4.75009	4.7531	1.10288	0.409	Sub	0.1359	0.5955	0.1126
5196	4.13830	4.07777	1.28685	0.91563	4.04256	-	1.49858	0.22396	Sub	0.0952	-	0.0938
5197	1.81296	1.71428	1.5543	0.67462	1.57987	-	1.10412	0.42773	Sub	0.2360	-	0.3697
5198	4.70422	4.48915	0.49756	0.22981	4.46588	4.4667	0.37613	0.68866	Sub	0.1376	0.5913	0.0734
5199	1.48485	1.17614	1.07484	0.31344	0.91221	-	0.513	0.93397	Sub	0.3881	-	0.4081
5200	0.48798	0.26932	1.62435	0.29477	0.29477	-	0.48244	1.54766	Free	0.4824	-	0.3627

Table 2. Results of generated discharge (q) in this study and comparison with other researchers (q_H =Henderson, 1966; q_R=Rajaratnam and Subramanya, 1967; q_S=Swamee, 1992).

No.	q	q_H	q_R	q_S	No.	q	q_H	q_R	q_S
1	1.221862	1.221862	1.162262	1.172048	19	0.725678	0.725678	0.69706	0.673461
2	1.631237	1.631237	-	1.762371	20	1.648285	1.648285	-	1.776251
3	1.378895	0.486694	-	0.363135	21	0.871679	0.871679	0.83958	0.677831
4	0.534783	0.534783	0.515934	0.347832	22	0.465574	0.465574	0.446378	0.420477
5	1.259101	1.469197	1.420665	1.389919	23	0.639441	0.639441	0.616846	0.387148
6	0.362973	0.362973	0.350221	0.226457	24	0.119889	0.119889	0.114086	0.119269
7	1.252711	1.252711	1.202455	1.159951	25	1.322795	1.322795	1.274138	1.083841
8	0.110771	0.110771	0.10683	0.035462	.	.	.	.	.
9	1.282546	1.282546	-	1.363531	.	.	.	.	.
10	1.995553	1.995553	-	2.386748	.	.	.	.	.
11	0.164335	0.164335	0.158481	0.073358	5193	1.030288	1.030288	-	1.041033
12	0.513341	0.513341	0.494724	0.416649	5194	1.801373	1.801373	1.729075	1.652081
13	1.713305	2.002556	1.936652	1.914865	5195	1.483729	1.483729	1.423835	1.229184
14	0.808257	0.808257	0.764728	0.762703	5196	1.286855	1.286855	-	1.26754
15	1.39896	2.220054	-	2.061595	5197	1.5543	1.5543	-	2.434875
16	0.685307	0.685307	0.65887	0.621988	5198	0.497563	0.497563	0.480154	0.26551
17	0.168067	0.168067	0.16199	0.130921	5199	1.074849	1.074849	-	1.130247
18	0.189058	0.189058	0.18043	0.18452	5200	1.624357	0.720158	-	0.541539

Finally F is specified for both free and submerged conditions with the aid of linear and nonlinear regression techniques. It should be said, in this procedure C_dS is used because Swamee (1992) formulas are based on experimental studies of flow behavior through a sluice gate. Although, we can use C_dH because this method has low value of MAPE, but this method is theoretical and it just has good accuracy comparing with Henry's curve for low values of y_1/b especially in submerged condition. So Swamee (1992) method is preferred.

Regression Analysis

Multiple regression analysis was carried out with different combinations of the dimensionless parameters in Eq. (17). Several linear and nonlinear multiple regressions were conducted using the Linear and Nonlinear Regressing Package of Mathematica v.6. The results for each flow condition are as follows.

a) Free Flow

The fitted linear and nonlinear equations for free flow and their determination coefficients are given by Eq. (21) and (22) respectively.

$$C_d = 0.4556 + 0.01194\left(\frac{y_1}{b}\right) - 0.000085\left(\frac{1}{Fr^2}\right), \; R^2 = 0.514 \quad (21)$$

$$C_d = 0.4445\left(\frac{y_1}{b}\right)^{0.1289}\left(\frac{1}{Fr^2}\right)^{0.0107}, \; R^2 = 0.806 \quad (22)$$

It can be seen from Eq. (21) and (22) that F_r is less important than y_1/b, so it can be canceled from regressing procedure to simplifying equations as following

$$C_d = 0.4552 + 0.01197\left(\frac{y_1}{b}\right), \; R^2 = 0.513 \quad (23)$$

$$C_d = 0.44457\left(\frac{y_1}{b}\right)^{0.1219}, \; R^2 = 0.7894 \quad (24)$$

Determination coefficient of Eq. (23) and (24) are very close to that of Eq. (21) and (22). This indicates that the discharge coefficient has just influenced by upstream water depth. So it is recommended to use Eq. (23) and (24) because of their simplicity and ease of application. Also, nonlinear equation has more precision compared with linear form, so it is better to use nonlinear equation.

Fig. 2 depicts C_d variation against y_1/b and $1/Fr^2$ and quietly acknowledges the stated points. In fact, Fig. 2-a is the same Henry's curve. The interesting mater about Fig. 2-b is that hysteresis phenomenon exists in data point trend. It means for a constant discharge rate, C_d is not same for increasing and decreasing flow rates. This had not addressed by other researchers previously.

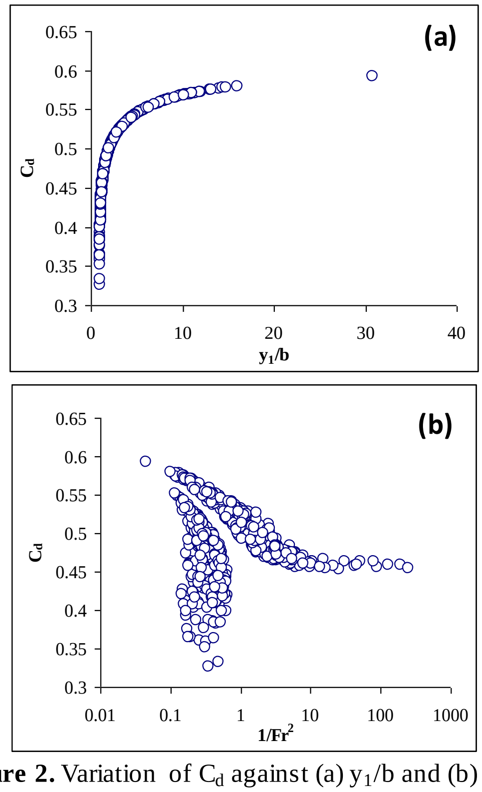

Figure 2. Variation of C_d against (a) y_1/b and (b) $1/Fr^2$ for free flow condition.

b) Submerged Flow

Multiple regression analysis was performed in submerged flow with different combinations of the dimensionless parameters y_1/b, y_3/b, y/b and $1/Fr^2$. The perfect fitted equations are given by Eq. (25) and (26).

$$C_d = 0.2681 - 0.0015\left(\frac{y_1}{b}\right) + 0.0982\left(\frac{y_3}{b}\right) - 0.1018\left(\frac{y}{b}\right) - 0.0013\left(\frac{1}{Fr^2}\right)$$

, $R^2 = 0.7347$ (25)

$$C_d = 0.8275\left(\frac{y_1}{b}\right)^{0.077}\left(\frac{y_3}{b}\right)^{-0.9898}\left(\frac{y}{b}\right)^{0.1637}\left(\frac{1}{Fr^2}\right)^{-0.4132}$$

, $R^2 = 0.9883$ (26)

Similarly, for submerged flow we can omit some of these parameters to obtain simple equations with approximately same precision. Because four dependant parameters are contributing in this condition, thus there will be fourteen other combinations of these parameters. Among these combinations the simplest and the most accurate linear and nonlinear equations are Eq. (27) and (28), respectively.

$$C_d = 0.2663 + 0.0905\left(\frac{y_3}{b}\right) - 0.0961\left(\frac{y}{b}\right),$$

$R^2 = 0.71$ (27)

$$C_d = 0.7482\left(\frac{y_3}{b}\right)^{-0.6825}\left(\frac{1}{Fr^2}\right)^{-0.3929},$$

$R^2 = 0.9831$ (28)

As can be seen Eq. (28) could state flow passes through a submerged sluice gate with a fine precision. So it is clear to use nonlinear equation for submerged condition, too.

In Fig. 3 variation of C_d with y_1/b, y_3/b, y/b and $1/Fr^2$ are presented. As it can be seen, discrepancy between these dimensionless parameters and C_d is too high (no trend line can be drawn between them), but interaction of these parameters with each other results high accuracy in prediction of C_d e.g. in Eq. (26).

In order to comparing the results of this study with other experimental studies, another simplified form of Eq. (26) is presented with independent parameters of y_1/b, y_3/b as Eq. (29).

$$C_d = 0.3865\left(\frac{y_1}{b}\right)^{1.0676}\left(\frac{y_3}{b}\right)^{-1.4486}, \quad R^2 = 0.82$$

As can be seen, Eq. (29) has less precision with respect to Eq. (28), but by the use of this equation we can make a good judgment about this study and other researchers. Fig. 4 and 5 present C_d prediction in free and submerged flow condition by Eq. (24) and (29), respectively with those of Henry (1950) and Swamee (1992).

It is interesting to note that, Eq. (24) and (29) have good clearance to Henry's curves and sometimes have more conformity compared with Swamee (1992) formulas results. Also, Eq. (24) and (28) or (29) are simpler than Eq. (10) and (11).

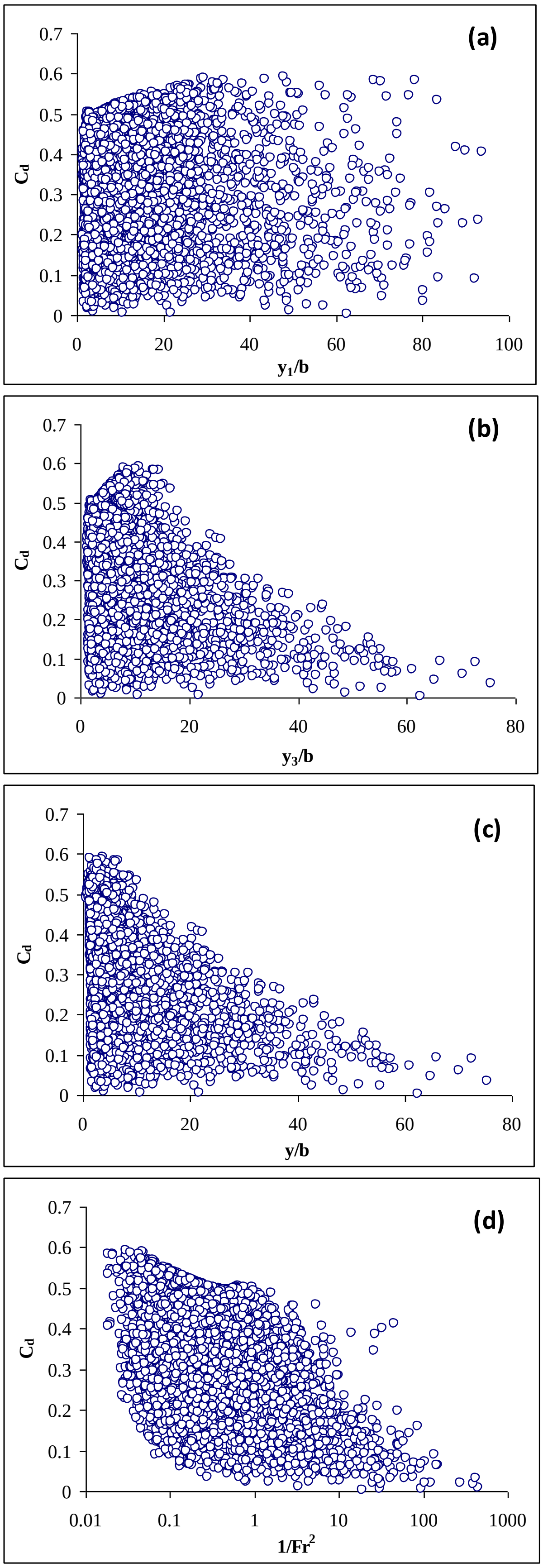

Figure 3 Variation of C_d against (a) y_1/b, (b) y_3/b, (c) y/b and (d) $1/Fr^2$ for submerged flow condition.

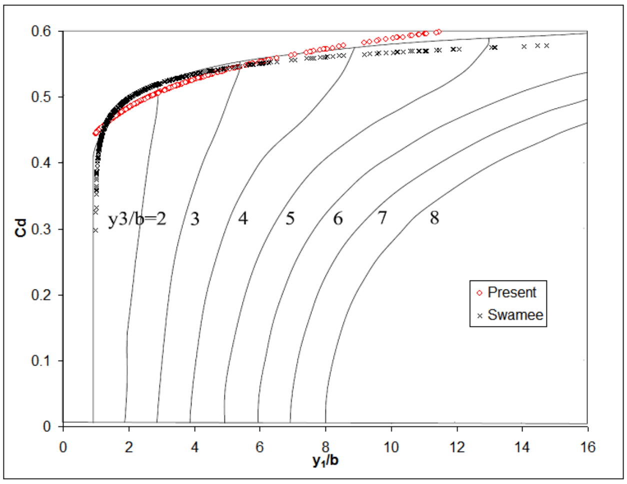

Figure 4. Prediction of C_d in free flow condition with those of Henry (1950) and Swamee (1992).

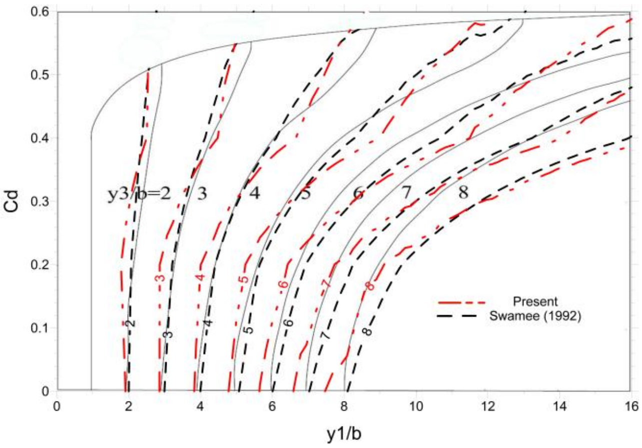

Figure 5. Prediction of C_d in submerged flow condition with those of Henry (1950) and Swamee (1992).

Finally, using Eq. (24) and (28), discharge coefficient for generated data points are computed. Then using Eq. (3) discharge rate obtained and MAPE were calculated for these values. Fig. 6 depicts MAPE values for these formulas and other researcher's formulas to calculate discharge coefficient. As one can see for present study MAPE equals to 21.54 % and this demonstrates its conformity with Swamee (1992) and improved value than Swamee (1992).

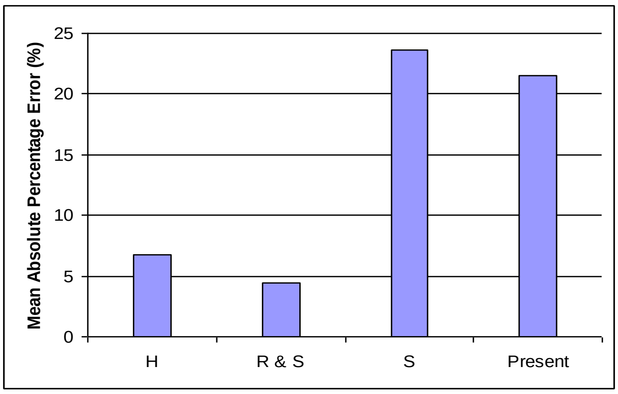

Figure 6. Comparison of MAPE for C_d obtained in present study and in Henderson, 1966 (H), Rajaratnam and Subramanya, 1967 (R & S), and Swamee, 1992 (S).

CONCLUSIONS

Study of free-surface flow under sluice gate is important to provide a prediction tool for the optimal management of irrigation and drainage channels. Flow through the gate may be free or submerged depending on the tailwater depth. Here, we considered an alternative to solve the governing equations. Our approach is based on solving Bernoulli's and specific force equations simultaneously with Wolfram Mathematica v.6 software. High quantity of data points (about 5200) in dimensionless form was produced. We compared the predictions obtained from numerical simulation and experiments performed on a laboratory by other researchers. Results showed high accuracy of present method in estimation of discharge coefficient. Effect of different parameters on estimation of discharge coefficient is shown by accurate regression equations.

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