

The Presence of Chaotic Behavior in Monthly Soil Temperature Time Series by Correlation Dimension Method

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ABSTRACT: Soil temperature can fluctuate dynamically, suggesting a possible chaotic behavior in such time series, as this can have a significant bearing on hydrological models, solar energy and other agricultural applications. Hence, this paper reports an investigation into the detection of a possible existence of chaotic behavior in the monthly soil temperature time series at five depths (5, 10, 20, 50 and 100 cm) below the ground level in a recorded data (an observation station in Adana, Turkey). For this purpose, the correlation dimension method, as a customary indicator of chaotic behavior, was derived from the data record over a period of 8 years (January 2000 - December 2007). The low correlation dimensions at five depths suggest the presence of low-dimensional chaotic behavior in the soil temperature dynamics.

Keywords: Chaos theory, Correlation dimension, Time series, Soil temperature

ORIGINAL ARTICLE

INTRODUCTION

An investigation is reported in this paper on the possible presence of chaotic behavior in monthly soil temperature time series with a focus on modeling studies but direct applications to project works is outside the remit of this paper. However, the motivation on the choice of soil temperature is that it plays a significant role in the hydrological models, solar energy applications and frost prediction (George, 2001; Mihalakakou, 2002; Koçak et al., 2004; Gao et al., 2008; Yılmaz et al. 2009).

The effects of soil temperature on agriculture are widespread and examples include the influence on seed germination (Nabi and Mullins, 2008), plant growth (Liu and Huang, 2005), uptake of nutrients (Dong et al. 2005), soil evaporation (Katul and Parlange 1992). The modeling strategies already employed include analytical models (e.g. Droulia et al., 2009), semi-analytical models (e.g. Kang et al. 2000), empirical models (e.g. Paul et al. 2004), Artificial Intelligence (AI) techniques (e.g. George, 2001; Bilgili 2010, 2011), numerical models (e.g. Gao et al., 2007 and 2008) and experimental methods (e.g. Enrique et al., 1999).

Recently, research on non-linear dynamics (deterministic chaos theory) has been applied to a wide range of practical problems as a modeling strategy, signifying a possible a loss of temporal correlation in response to small perturbations, particularly in initial conditions. The application areas of chaos theory include many areas of natural sciences e.g. hydrology, hydrometeorology, oceanography (e.g. Sivakumar et al., 2006; Henderson and Wells 1988; Khan et al. 2005; Men et al., 2004; Sivakumar, 2001 and 2000; Khatibi et al., 2010; Ghorbani et al., 2011; Sivakumar et al., 1999; Stehlik, 1999 and many others).

Soil temperature fluctuations are driven by variations in air temperature and solar radiation and the

effects are displayed in daily and annual timescales. Theoretical models of the annual variation of daily average soil temperature are expressed by using a sinusoidal function at different depths (Hillel, 1982; Marshall and Holmes, 1988; Wu and Nofziger, 1999), reminiscent of tidal models. However, diurnal soil temperature fluctuations are functions of time and depth and Paul et al. (2004) argue that the fluctuations further depend on such parameters as: precipitation, soil texture, and moisture content as well as the type of surface cover (plant canopy, crop residue, snow. Thus, at daily timescales soil temperature fluctuations can be dynamic, particularly at the topsoil strata, as opposed to substrate soil temperature. The simple proposition in this paper is therefore to test if soil temperature can be modeled by chaos theory, which is a reflection of internal behaviors in the time history of one (or more) of system variables, normally referred to as time series.

Chaos theory has been applied widely including those to soil science e.g. soil system (Culling 1988), ecological data (Turching and Taylor, 1993), soil formation (Phillips 1998), gold grade spatial series (Xie and Chen 2004), paddy soil strength (Lu Zhi-Xiong and Pan Jun-zheng, 2000), near-surface temperature (Koçak et al., 2004), electrical conductivity and gravimetric water content (Millán et al., 2009). However, the authors are not aware of any study exploring possible presence of chaotic behaviors in soil temperature and hence this study. There are various methods for identification of chaotic behavior and this study selects the simple correlation dimension method to detect the chaotic behavior in the time series (Zhi-Xiong and Jun-zheng 2000). The possible presence of chaotic behavior in monthly soil temperature time series is investigated at five depths (5, 10, 20, 50 and 100

cm) below the ground level over a period of 8 years (January 2000 - December 2007) in Adana, Turkey.

The time series investigation reported in this paper is motivated by seeking modeling strategies to practical problems. In general, two broad approaches are feasible: (i) distributed models, which are based on the application of laws of nature or empirical techniques (this approach is not used in this paper); and (ii) local models, which are often bottom-up data-driven techniques seeking deriving appropriate information from recorded data. Chaos theory is one of local modeling strategies and applied here to soil temperature. Practical implications of presence of chaotic signals in soil temperature are discussed later in the paper. The remainder of the paper is organized as follows. Section 2 presents the methodology in this study. In Section 3, the data used, case study and results obtained are explained. The conclusions of this study are presented in Section 4.

MATERIAL AND METHODS

Several methods are available for investigating chaotic behaviors in time series. The correlation dimension method suggests the possible presence of chaotic behavior and fractal characteristics of a process, leading to the identification of the number of dominant variables present in the evolution of the corresponding dynamical system (Sivakumar 2001). Several methods have been suggested to calculate the correlation dimension of the time series data (Grassberger and Procaccia 1983; Theiler et al. 1992). The algorithm of Grassberger and Procaccia (1983) is the most commonly used algorithm, which is used in this study to compute the correlation dimension of soil temperature time series. The algorithm is based on the phase space reconstruction of the time series. In this method a scalar time series $(x_1, x_2, x_3, \dots, x_N)$ is first considered.

According to the Takens embedding theorem (Takens 1981), an m -dimensional phase space can be reconstructed from the time series X_i of soil temperature variations as:

$$X_i = (x_i, x_{i+\tau}, x_{i+2\tau}, \dots, x_{i+(m-1)\tau}) \quad (1)$$

where m is called *embedding dimension* and τ is referred to as *delay time*. Phase space reconstruction requires an optimum delay time. Many methods have been proposed for the estimation of optimal values of the delay time. The Average Mutual Information (AMI) is used in this study for τ , as suggested by (Frazer and Swinney, 1986).

For a given time series sequence $\{x_0, x_1, x_2, \dots, x_i, \dots, x_n\}$ the mutual information indicates the amount of information about the state $x_{i+\tau}$ if the state of x_i is known. The average mutual information is defined by:

$$I(\tau) = \sum_{i, i+\tau} p(x_i, x_{i+\tau}) \ln \frac{p(x_i, x_{i+\tau})}{p(x_i) \cdot p(x_{i+\tau})} \quad (2)$$

Where $p(x_i)$ and $p(x_{i+\tau})$ correspond to the probability of finding x_i in the i th and $x_{i+\tau}$ in the j th interval, and $p(x_i, x_{i+\tau})$ is their joint probability.

The first local minimum of $I(\tau)$ estimates the optimal selection for the delay time required for phase space reconstruction. In practice, it provides the maximum delay time such that $x_{i+\tau}$ adds the largest amount of information about x_i .

For an m -dimensional phase space, the correlation dimension is computed in terms of (Fraser and Swinney 1986):

$$C_m(r) = \lim_{N \rightarrow \infty} \frac{2}{N(N-1)} \sum_{i, j=1; i \neq j}^N H(r - \|X_i - X_j\|) \quad (3)$$

where $H(x)$ is the Heaviside step function defined as $H(x)=1$ for $x>0$ and $H(x)=0$ for $x \leq 0$, X_i and X_j can be obtained using Eq.(3), $\|X_i - X_j\|$ is the Euclidean distance between X_i and X_j , N is the number of points on the reconstructed attractor and r is the radius of the sphere centered on X_i or X_j .

For stochastic time series $C_m(r) \propto r^m$ holds, whereas for chaotic time series the correlation function scales with r as:

$$C_m(r) \propto r^{D_2} \quad (4)$$

where D_2 , called *correlation exponent*. The correlation exponent is defined by

$$D_2 = \lim_{r \rightarrow 0} \frac{\ln C_m(r)}{\ln r} \quad (5)$$

and can be reliably estimated as the slope in the $\ln C_m(r)$ vs. $\ln(r)$ plot. The slope can be computed by the least-squares fit of a straight line over a length scales of r .

The average mutual information (AMI) approach and the correlation integral analysis are implemented by TISEAN package (Hegger et al., 1999).

According to Grassberger-Procaccia algorithm (1983), the behavior of ' m ' versus ' D_2 ' may undergo the following cases, which are illustrated in Figure 1: (i) for deterministic dataset, the behavior should be a straight line parallel to embedding dimension; (ii) for stochastic dataset, it should be straight line sloping 45 degrees to x and y axis; (iii) for chaotic system, the correlation exponent initially increases but finally saturates after a special embedding dimension. The saturation value of the correlation exponent is defined as the correlation dimension. In the above cases (stochastic, deterministic, chaotic), the behavior is a low-dimensional chaos, if the value of correlation dimension is small and fractal.

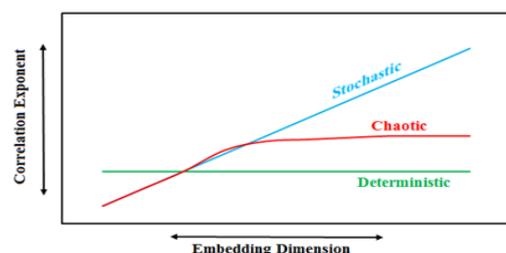


Figure 1. Plot differentiating deterministic, stochastic and chaotic system

RESULTS AND DISCUSSION

The monthly soil temperature data at five depths (5, 10, 20, 50 and 100 cm) over a period of 8 years (January 2000 - December 2007) is used in the study. The data were obtained from Adana meteorological station, located at 36°59'N, 35°18'E (Figure 2) at an altitude of 28 m above sea level in the eastern Mediterranean region of Turkey. Adana is the fourth largest city of Turkey, and it is a major agricultural and commercial center (Bilgili 2010). The statistical parameters of the soil temperature data for the Adana meteorological station are given in Table 1 and Figure 3 shows the variations of monthly data series.



Figure 2. The location of Adana meteorological station in Turkey (Bilgili 2010)

Table 1. Statistics of monthly soil temperature time series for all depths

Soil depth (cm)	Number of data	Mean (°C)	Standard deviation (°C)	Maximum value (°C)	Minimum value (°C)	Variance (°C ²)	Skewness
5	96	22.17	9.61	37.6	7.1	92.41	0.051
10	96	21.68	8.89	36.2	7.8	79.20	0.019
20	96	21.40	8.36	34	8.5	69.94	0.002
50	96	21.39	7.44	32.8	10.4	55.36	0.028
100	96	21.44	6.09	30.9	12.3	37.12	0.059

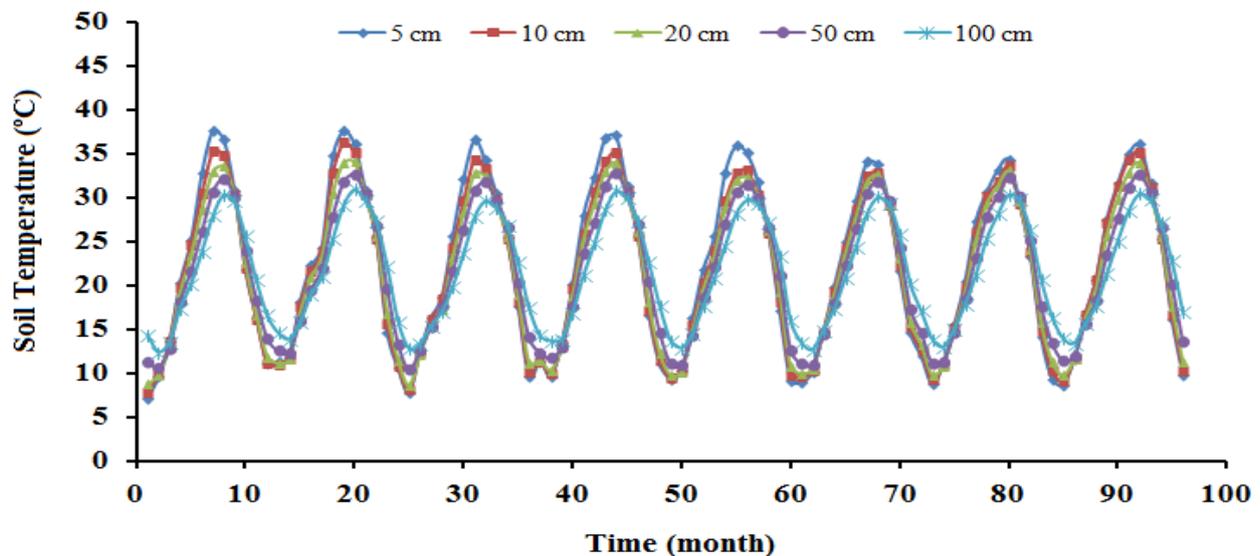


Figure 3. Monthly variations of soil temperature for all depths (January 2000 - December 2007)

To compute the correlation dimension, the delay time or τ was computed using the AMI method for different lag times (Figure 4). The first minima in the mutual information function can be considered as the optimal delay time. Hence, the optimal delay time is chosen as 4, 2, 2, 2 and 2 months for 5, 10, 20, 50 and 100 cm depth of soil, respectively.

The correlation function is calculated for the dataset using the delay times ($\tau=4, 2, 2, 2$ and 2), determined by the AMI method, and for embedding dimensions, m , from 1 to 30. Figure 5(a-e) shows the relationship between the correlation function $C(r)$ and the radius r (i.e. $\ln C(r)$ versus $\ln r$) for increasing m for all depths. The relationship between the correlation dimension values $D_2(m)$ and the embedding dimension values m is shown in Figure 5(f-j).

Figure 5(f-j) shows that the value of correlation exponent increases with the embedding dimension up to a

certain value and then saturates beyond it. The saturated correlation dimension (D_2) and the minimum number of required variables to describe the dynamics of soil temperature for all depth listed in Table 2. The low and non-integer dimension suggests the presence of chaotic behavior and low dimensional deterministic dynamics in the soil temperature at all depths. To describe the dynamics of the soil temperature variations, the minimum number of variable is required. According to correlation dimension concepts, the nearest integer above the correlation dimension value provides the number of dominant variables influencing the dynamics of the underlying system (Sivakumar 2000). The correlation dimensions for the monthly soil temperature at all depths are 1.6, 1.8, 1.6, 1.4 and 1.6, so at least 2 independent variables are needed to describe the dynamics of the soil temperature variation, respectively.

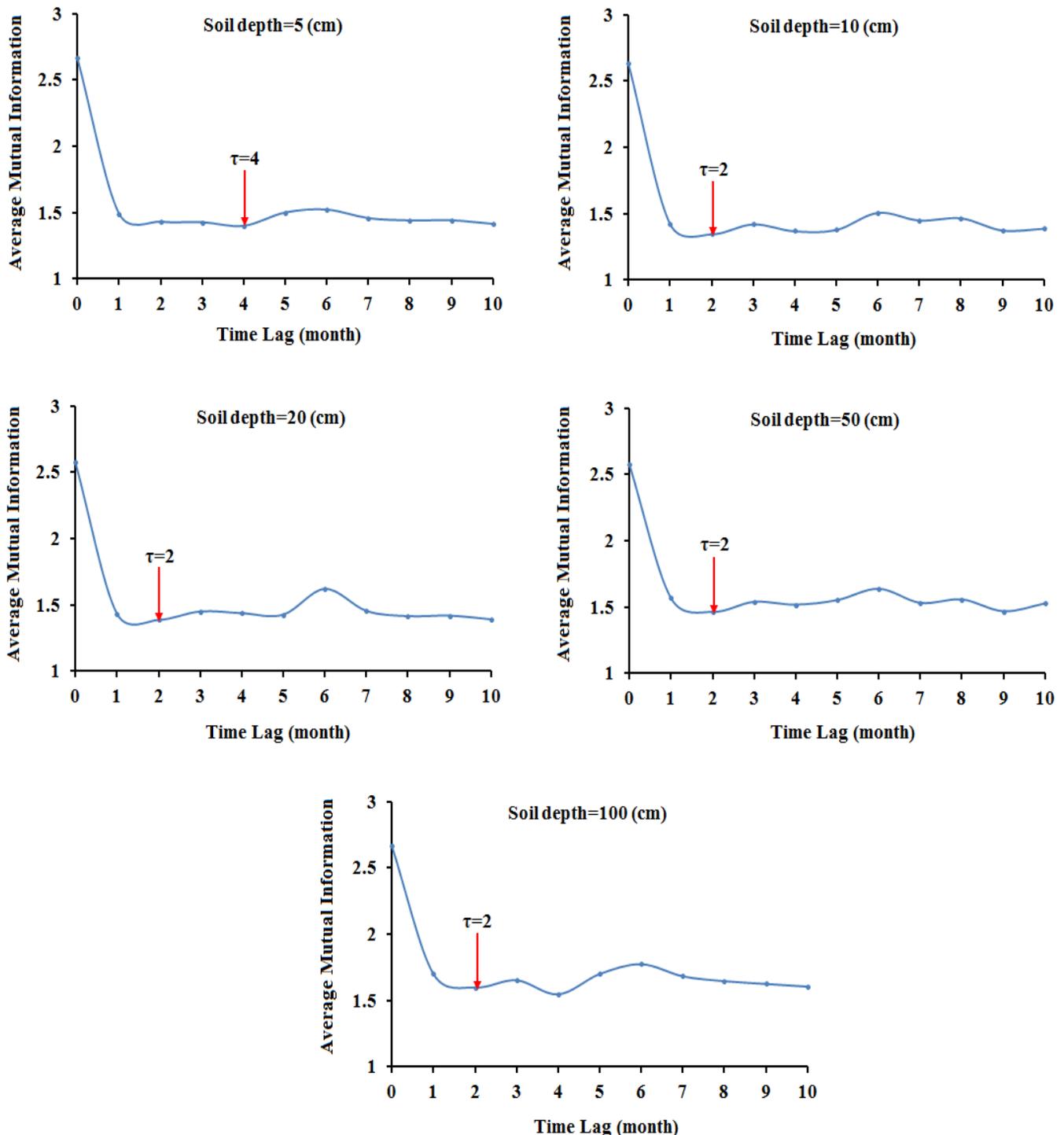


Figure 4. Average Mutual Information (AMI) function of the soil temperature for all depths

The fundamental concept in chaos theory is that complex and random-looking behaviors are not necessarily the result of actual random systems but can also be from simple nonlinear deterministic systems with sensitive dependence on initial conditions. The derived parameters in Table 2 define the nature of chaotic signals in the dataset to be low dimensional. With this knowledge in hand, predicting future values become feasible, which can serve many practical problems such as soil temperature in hydrology, frost prediction and various applications in agriculture.

This study is concerned only with the identification of chaotic signals, which can easily be extended as a prediction/forecast tool, for more details on prediction,

see Sivakumar (2002), Koçak (1997) and Khatibi (2012), among others. While the purpose of using chaos theory should be to explain the physical mechanisms of the underlying system dynamics triggering chaotic behavior, applications so far have largely focused on identification and prediction of time series only. An extension to this study can be the identification of physical conditions under which low dimensional chaotic behavior prevails.

This area of chaotic behaviors is seemingly overlooked and the authors are now focused on identifying telltale signs in physical behavior that are preserved in the models causing a loss of temporal correlation in response to small perturbations, often driven by initial conditions.

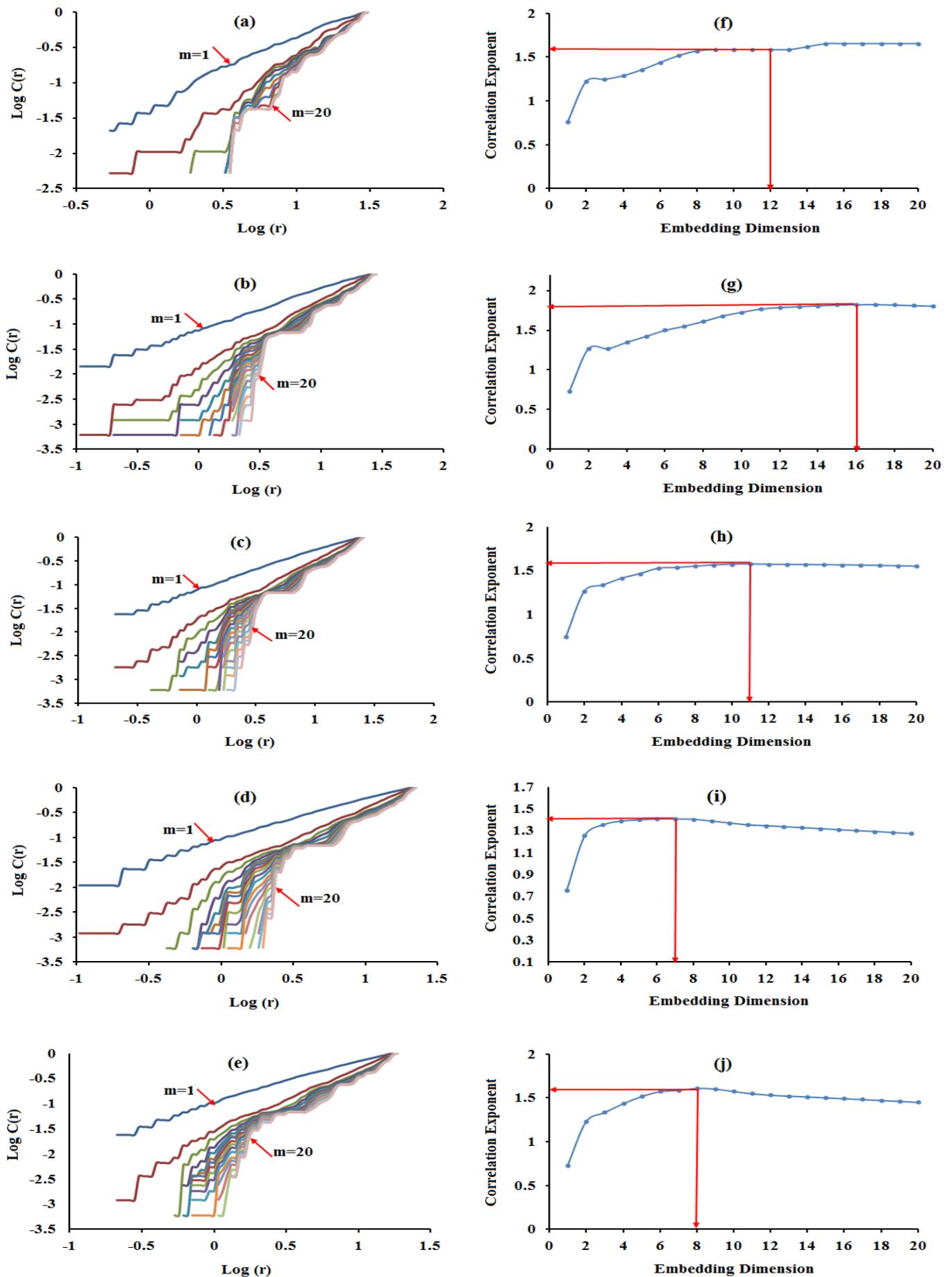


Figure 5. Correlation dimension results for soil depth time series: (a)–(e) $\log C(r)$ vs $\log r$ relationship at different depths; and (f)–(j) correlation exponent vs embedding dimension at different depths – (a,f) depth=5 cm, (b,g) depth=10 cm, (c,h) depth=20 cm, (d,i) depth=50 cm, (e,j) depth=100 cm

Table 2. The correlation dimensions of soil temperature at all depths

Soil depth (cm)	Correlation dimension (D_2)	Maximum number of required variables to describe the dynamics of soil temperature
5	1.6	2
10	1.8	2
20	1.6	2
50	1.4	2
100	1.6	2

CONCLUSIONS

The existence of low dimensional chaotic behavior is investigated in the monthly soil temperature data at five depths 5, 10, 20, 50 and 100 cm in a gauging station in the city of Adana, Turkey. The average mutual information (AMI) approach and the correlation integral analysis were used in the research by TISEAN package (Hegger et al. 1999). The mutual information approach provided a time lag which is needed to estimation of correlation dimension values. The low and non-integer dimension suggests the presence of chaotic behavior and low dimensional deterministic dynamics in the soil temperature at all depths. Based on the correlation dimensions, the minimum numbers of variables essential to model the monthly soil temperature dynamics were identified, signifying a more predictable model for predicting future values of dynamic soil temperature values in the presence of chaotic behavior.

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REFERENCES

- Bilgili M (2010). Prediction of soil temperature using regression and artificial neural network models. *Meteorol Atmos Phys.* 110:59–70.
- Bilgili M (2011). The use of artificial neural networks for forecasting the monthly mean soil temperatures in Adana, Turkey. *Turk J Agric For* 35, 83-93.
- Culling WEH (1988). Dimension and entropy in the soil-covered landscape. *Earth Surf.Process. Landf.* 13, 619–648.
- Dong S, Scagel CF, Cheng L, Fuchigami LH, Rygiewicz PT (2001). Soil temperature and plant growth stage influence nitrogen uptake and amino acid concentration of apple during early spring growth. *Tree Physiology.* 21(8) 541–547.
- Droulia F, Lykoudis S, Tsiros I, Alvertos N, Akyilas E, Garofalakis I (2009). Ground temperature estimations using simplified analytical and semi-empirical approaches. *Sol Energy* 83:211–219.
- Enrique G, Braud I, Jean-Louis T, Michel V, Pierre B, Jean- Christophe C (1999). Modelling heat and water exchanges of fallow land covered

- with plant-residue mulch. *Agric For Meteorol* 97:151–169.
- Fraser AM, Swinney HL (1986). Independent coordinates for strange attractors from mutual information. *Phys Review A* 33(2):1134-1140.
- Gao Z, Bian L, Hu Y, Wang L, Fan J (2007). Determination of soil temperature in an arid region. *J Arid Environ* 71:157–168.
- Gao Z, Horton R, Wang L, Liu H, Wen J (2008). An improved forcerstore method for soil temperature prediction. *Eur. J Soil Sci.* 59:972–981.
- George RK (2001). Prediction of soil temperature by using artificial neural networks algorithms. *Nonlinear Anal* 47:1737–1748.
- Ghorbani MA, Kisi O, Aalinezhad M (2010). A probe into the chaotic nature of daily streamflow time series by correlation dimension and largest Lyapunov methods. *Applied Mathematical Modelling* 34, 4050–4057.
- Grassberger P, Procaccia I (1983). Characterization of strange attractors. *Phys Rev Lett* 50: 346-349.
- Hegger R, Kantz H, Schreiber T (1999). Practical implementation of nonlinear time series methods: The TISEAN package. *Chaos.* 9:413-435.
- Henderson HW, Wells R (1988) Obtaining attractor dimensions from meteorological time series. *Adv Geophys* 30: 205–237.
- Kang S, Kim S, Oh S, Lee D (2000). Predicting spatial and temporal patterns of soil temperature based on topography, surface cover and air temperature. *For Ecol Manag* 136:173–184.
- Katul GG, Parlange MB (1992). Estimation of bare soil evaporation using skin temperature measurements. *Journal of Hydrology.* 132(1-4) 91–106.
- Khan S, Ganguly AR, Saigal S (2005). Detection and predictive modeling of chaos in finite hydrological time series. *Nonlinear Process Geophys* 12:41–53.
- Khatibi R, Ghorbani MA, Aalami MT, Kocak K, Makarynskyy O, Makarynska D, Aalinezhad M (2011) Dynamics of hourly sea level at Hillarys Boat Harbour, Western Australia: a chaos theory perspective. *Ocean Dynamics* 61:1797–1807.
- Khatibi R, Sivakumar B, Ghorbani MA, Kisi O, Kocak K, and Farsadi Zadeh D (2012).
- Investigating chaos in river stage and discharge time series. *J Hydrol* 414–415: 108–117. <http://dx.doi.org/10.1016/j.jhydrol.2011.10.026>
- Koçak K (1997) Application of local prediction model to water level data. A satellite Conference to the 51 st ISI Session in Istanbul, Turkey. *Water and Statistics, Ankara-Turkey* 185-193
- Liu X, Huang B (2005). Root physiological factors involved in cool-season grass response to high soil temperature. *Environmental and Experimental Botany.* 53(3) 233–245.
- Men B, Xiejing Z, Liang C (2004) Chaotic Analysis on Monthly Precipitation on Hills

- Region in Middle Sichuan of China. *Nat Sci* 2:45–51.
24. Millán H, García-Fornaris I, González-Posada M (2009) Nonlinear spatial series analysis from unidirectional transects of soil physical properties. *Catena* 77, 56–64.
 25. Nabi G, Mullins CE (2008) Soil temperature dependent growth of cotton seedlings before emergence. *Pedosphere*, 18(1) 54–59.
 26. Packard NH, Crutchfield JP, Farmer JD, Shaw RS, (1980) Geometry from a time series. *Phys. Rev. Lett.* 45, 712–716.
 27. Paul KI, Polglase PJ, Smethurst PJ, O’Connell AM, Carlyle CJ, Khanna PK (2004) Soil temperature under forests: a simple model for predicting soil temperature under a range of forest types. *Agric For Meteorol* 121:167–182.
 28. Phillips JD, (1998) On the relation between complex systems and the factorial model of soil formation (with discussion). *Geoderma* 86, 1–42.
 29. Sivakumar B (2000) Chaos theory in hydrology: important issues and interpretations. *J Hydrol* 227:1–20.
 30. Sivakumar B (2001) Rainfall dynamics in different temporal scales: a chaotic perspective. *Hydrol Earth Syst Sci* 5:645–651.
 31. Sivakumar B, Berndtsson R, Lall U (2006) Nonlinear deterministic dynamics in hydrologic systems: present activities and future challenges. *Nonlinear Process Geophys Preface* 1–2.
 32. Sivakumar B, Liang SY, Liaw CY, Phoon KK (1999) Singapore rainfall behavior: Chaotic. *J Hydrol Eng ASCE* 4:38–48.
 33. Sivakumar B, Jayawardena AW, Fernando TMGH (2002a) River flow forecasting: Use of phase space reconstruction and artificial neural networks approaches. *J Hydrol* 265(1-4): 225-245
 34. Stehlik J (1999) Deterministic chaos in runoff series. *J Hydrol Hydromechanics* 47:271–287.
 35. Takens F (1981) Detecting strange attractors in turbulence, in *Lectures Notes in Mathematics*, edited by D.A.Rand and L.S.Young, Springer-Verlag, New York. 898:366-381.
 36. Theiler J, Eubank S, Longtin A, Galdrikian B, Farmer JD (1992) Testing for nonlinearity in time series: the method of surrogate data. *Physica D* 58, 77–86.
 37. Turching P, Taylor AD (1993) Complex dynamics in ecological time series. *Ecology* 73,289–305.
 38. Xie, YS, Chen GH (2004) Chaotic analyses for space series of gold grade. *Int. J. Mod.Phys. B* 18, 2730–2733.
 39. Yilmaz T, Ozbek A, Yilmaz A, Buyukalaca O (2009) Influence of upper layer properties on the ground temperature distribution. *J Therm Sci Technol* 29:43–51.
 40. Zhi-Xiong L, Jun-zheng P (2000). Correlation dimension of paddy soil strength in China. *Journal of Terramechanics* 37, 185-189.