

An Investigation of Efficiency and Accuracy of Energy-Based Capacity Spectrum in Estimating the Seismic Demands of Buildings

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ABSTRACT: In the conventional methods of pushover analysis, the capacity spectrum is obtained with linking roof displacement to base shear, while this relationship only is meaningful in pushover analysis with load pattern that is proportional to the first mode. In the higher modes, changes of the roof displacement is not proportional to the changes of other stories displacement, and even in some cases roof displacement is decreased with increasing base shear and reverses. Therefore, using the roof displacement for establishing the capacity spectrum in the higher modes is faced with ambiguities. Using the energy method for establishing the capacity spectrum of structure is one of the proposed methods to eliminate these ambiguities. In this method, not only the amount of the roof displacement, but also the amount and the sign of other stories displacements participate in the establishment of capacity spectrum of structure. In this research, the accuracy and efficiency of energy-based capacity spectrum has been studied on two steel moment-frame buildings under different earthquakes for using in different pushover analysis with various load patterns. The results show that using the energy method for establishing the capacity spectrum not only eliminate the existent ambiguities and problems in establishment of capacity spectrum, but also the responses of the different nonlinear static analysis by using the energy-based capacity spectrum instead of the conventional capacity spectrum (based on roof displacement) have better accuracy and efficiency.

Keywords: Seismic Demands, Modal Pushover Analysis, Higher Modes, Capacity Spectrum, Energy Method

ORIGINAL ARTICLE

INTRODUCTION

According to the concept of seismic design of structures and their nonlinear behaviour at low performance levels, it is clear that the vulnerability of structures against earthquake is controlled by deformation capacity of inelastic structural elements. Hence, the change of basis of regulations from force control mode to the displacement control mode has been recommended by various researchers, and this requires the use of nonlinear analysis (Applied Technology Council, 1996; European Committee for Standardization, 2002; Otani et al., 2000). Considering the complexity of nonlinear dynamic analysis, pushover analysis has been developed as a practical tool for estimating the seismic demands of inelastic structures. The conventional methods of pushover analysis in the current documents and instructions such as ATC-40 and FEMA-356 have been faced with the shortcomings in comparison with nonlinear dynamic analysis. In the conventional pushover analysis, a monotonic lateral force pattern is applied to the structural model in a step-by-step static analysis until a target displacement is achieved. Since in the low-rise buildings the first mode is predominant, this procedure accurately can estimate the global seismic responses. But, in high-rise and irregular buildings where the effects of the higher modes are important, this procedure is defective (Krawinkler and Seneviratna, 1998; Kim and

D'Amore, 1999; Mwafy and Elnashai, 2001; FEMA, 2005). However, with several practical developed solutions, accuracy and efficiency of these methods can significantly be improved. Therefore, during the past decade various solutions and advanced pushover analysis methods have been proposed.

In order to incorporate the effects of the higher modes, some advanced modal pushover procedures based on the elastic modal decomposition concepts have been developed that maintain the simplicity of conventional pushover methods (Paret et al., 1996; Sasaki et al., 1998; Moghadam, 2002; Chopra and Goel, 2002; Chopra et al., 2004; Mori et al., 2006; Shakeri et al., 2007). The modal pushover analysis (MPA) is the prominent proposed procedure that the multiple pushover analyses with a lateral load corresponding to the considered elastic mode shapes are conducted separately, and then the total seismic response is estimated by combining the responses due to each modal load (Chopra, and Goel, 2002). Also, a modified version of the MPA (MMPA) based on elastic spectral response was proposed (Chopra and Goel, 2004).

Since these multi-run methods are unable to reflect the yielding effect of one mode in the other modes and the interaction between modes in the nonlinear range, some researchers have developed enhanced single-run modal methods in which the structures are pushed with combined modal forces (Matsumori et al., 1999; Kunnath, 2004; Jan et al., 2004; Elnashai, 2001; Antoniou and Pinho, 2004;

Shakeri et al., 2009).

The modal combination concept such as square-root-of-the-sum-of-the-squares (SRSS) is used to define the load pattern rather than to combine the nonlinear responses due to each mode. In the applied load pattern the effects of higher modes and interaction between them are considered in inelastic area. Also, behaviour of the structure, sequence of plastic hinges occurrence, the local and global mechanisms during the analysis are pursued easily by single-run pushover analysis. In this regard, researchers such Elnashi proposed an advanced single-run pushover analysis method (Elnashai, 2001) and afterwards by Antoniou and Pinho through a fiber analysis model, called FAP method (Force-based Adaptive Pushover) was developed (Antoniou and Pinho, 2004). Also, in FFMA 356 to consider the effect of higher modes, a single-run modal pushover analysis method has been proposed that is called storey shear-based pushover (SSP) analysis. In this method the load pattern is derived from the modal story shears profile. For this purpose, the modal story shear in each mode is determined from the elastic spectrum and is combined together by using modal combination methods. Then, the modal shear profile is obtained. Applied load pattern is extracted from modal shear profile, so that the shear obtained from applied load pattern on each story is equal to its modal shear (American Society of Civil Engineers, 2000).

Since in the higher modes, the increase of the roof displacement is not proportional to the other stories displacement, and even may reverse, using the roof displacement for establishing the capacity spectrum in the higher modes is faced with ambiguities. In this regard, a procedure has been proposed based on the energy concepts for establishing the capacity. In this method not only the amount of the roof displacement, but also the amount and sign of other stories displacement participate in establishment of capacity spectrum of structure spectrum (Hernandez-Montes et al., 2004).

Energy-based capacity spectrum curve

In the conventional nonlinear static procedure, the lateral load pattern is defined based on the assumed constant fundamental mode shape using equation (1). Then the pushover curve of the multi-degree-of-freedom (MDOF) structure (base shear versus roof displacement) resulting from non-linear static analysis is converted to an acceleration versus spectral displacement (ADRS) curve or force versus displacement ($F^* - D$) curve of the equivalent SDOF system with unit mass by equations (2) and (3). The target displacement is estimated through the maximum inelastic displacement of the equivalent SDOF system.

$$\{f\} = [m] \times \{\varnothing\} \quad (1)$$

$$F^* = S_a = \frac{V_b}{M^*} \quad (2)$$

$$D^* = S_d = \frac{u_r}{\Gamma \varnothing_r} \quad (3)$$

where, $\{\varnothing\}$: is the assumed fundamental mode shape, $[m]$: the mass matrix, V_b : the base shear, u_r : the roof displacement, \varnothing_r : the component of the $\{\varnothing\}$ in the roof level, $\Gamma = (\{\varnothing\}^T [m] \{\varnothing\}) / (\{\varnothing\}^T [m] \{1\})$ the

participation factor, $M = \Gamma \cdot L$: the effective mass and $L = \{\varnothing\}^T [m] \{1\}$.

Since the conventional nonlinear static methods in the documents such as ATC-40 and FEMA-356 are limited to an assumed fundamental mode like first mode, the effects of higher modes are not considered. Therefore, the application of conventional nonlinear static methods in high-rise and irregular buildings does not have appropriate accuracy where the effects of the higher modes are important. In order to solve this problem, various modal pushover methods have been proposed. But, because of the assumed fundamental mode shape of modal pushover methods, the increase of the roof displacement is not proportional to the other stories displacement. Therefore, using the roof displacement as a conversion parameter from the MDOF system to the SDOF system cannot be meaningful except for the first mode. In previous section it was mentioned that the changes of the roof displacement in the higher modes are not proportional to the changes of other stories displacement. In some buildings in the higher modes after yielding and forming the plastic hinges, roof displacement is reduced with increasing base shear, and capacity curve moves conversely (As shown in the third mode of three-storey building SAC research team (Figure 1). So, the establishment of capacity curve for higher modes will be faced with the problem.

In order to solve this problem, an innovative method by Hernandez-Montes et al. has been proposed to convert the displacement coordinates of MDOF system to the displacement of equivalent SDOF system by using the concept of absorbed energy (or work done) at different stories. In this method not only the amount of the roof displacement, but also the effects of amount and sign of other stories displacement participate in determining the displacement of an equivalent SDOF system and establishing the capacity spectrum (Hernandez-Montes et al., 2004).

In this method, at each step of loading (k), the increment of the displacement in the SDOF system is defined based on the sum of the work done at the different story levels through each incremental force, $dF_i^{(k)}$. At each step, the total work done in all stories is assumed to be equal to the work done by the base shear (equation (4)), and the equivalent displacement is calculated using equations (5) and (6).

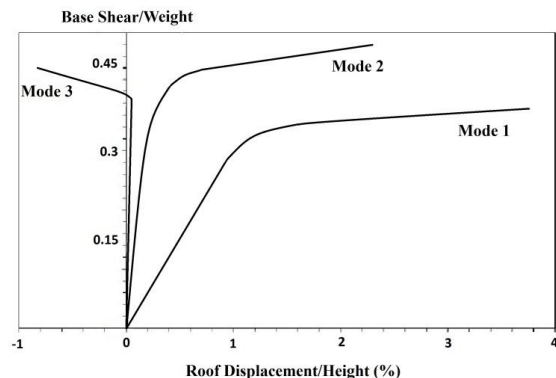


Figure 1. Pushover curves for three-storey steel frame (SAC 3) in the first three modes obtained from OPENSEES.

$$\sum_{i=1}^n \left(\left(F_i^{(k-1)} + \frac{1}{2} dF_i^{(k)} \right) \times \Delta d_i^{(k)} \right) = \left(\sum_{i=1}^n F_i^{(k-1)} + \frac{1}{2} \sum_{i=1}^n dF_i^{(k)} \right) \times \Delta D^{(k)} \quad (4)$$

$$\Delta D^{(k)} = \sum_{i=1}^n \left(F_i^{(k-1)} + \frac{1}{2} dF_i^{(k)} \right) \times \Delta d_i^{(k)} \bigg/ \sum_{i=1}^n \left(F_i^{(k-1)} + \frac{1}{2} dF_i^{(k)} \right) \quad (5)$$

$$D^{(k)} = D^{(k-1)} + \Delta D^{(k)} \quad (6)$$

where, $F_i^{(k-1)}$: is the existing force in the story i at the end of step $k-1$, $dF_i^{(k)}$: incremental applied force in the story i at step k , $\Delta d_i^{(k)}$: incremental displacement in the story i due to increased applied load at step k , $\Delta D^{(k)}$: incremental displacement of the equivalent SDOF system at step k , $D^{(k)}$: displacement of the equivalent SDOF system at step k .

At each step of loading, capacity spectrum curve ($F^* - D$) of equivalent SDOF system easily can be drawn by computing the equivalent force of SDOF system ($F^{*(k)}$) using equation (2), and computing the displacement of equivalent SDOF system ($D^{(k)}$) using equation (5).

As shown in Figure 2, the reversal apparent with the conventional roof-displacement approach in establishment of capacity spectrum curve is rectified when the energy-based approach is used.

An investigation of efficiency and accuracy of energy-based Capacity spectrum method

As described in the previous section, using concept of absorbed energy can eliminate the existent ambiguities and problems over establishment of capacity spectrum in higher modes. But, the basic question is whether energy-based capacity spectrum is efficient and accurate in using pushover analysis methods or not? To answer this question, all the modelling and analyzing processes carried out using OPENSEES computer program which has high ability in performing non-linear analysis. These structural models are subjected to the different ground motions and the peak responses resulting from the nonlinear time history (NTH) analysis are considered as benchmark responses. The structural models are subjected to the different nonlinear static procedures (NSPs). The intended results are gotten through two different approaches, i.e., capacity curve based on roof-displacement approach (conventional capacity curve) and energy-based capacity curve. Finally, the results obtained from both mentioned approaches are compared with those obtained from the NTH analysis.

Structural models

The studied structural models in this research are the typical medium and high-rise steel buildings which are known as SAC-9 and SAC-20 buildings (Gupta and Krawinkler, 1999). SAC-9 and SAC-20 buildings are respectively nine-story and twenty-story perimeter steel moment resistant frame (SMRF) buildings designed by consulting structural engineers for the Phase II of the SAC project. In the design of these structures, the seismic code requirements of the 1994 UBC for the Los Angeles area are considered. The SAC-9 and SAC-20 have been used by numerous researchers in recent years as benchmark structures (FEMA 440, 2005; Gupta and Krawinkler, 1999; Goel and Chopra, 2004). In this study, the two dimensional models are considered. Only one of the perimeter steel moment resisting frames in the N-S

direction with applying half weight of the building is modelled.

Ground motions

Typical Structures SAC-9 and SAC-20 have been studied under 10 near-fault ground motions. The main properties of the selected records are presented in Table 1. The records are available in the Pacific Earthquake Engineering Research (PEER) site, <http://peer.berkeley.edu/smcat>. The important objective in selection of the records is that they persuade typical buildings to enter in inelastic phase and excite the effect of the higher modes. Among the two horizontal components of a near-fault motion, the considered component is the one that possess the maximum ground velocity (Kalkan and Kunnath, 2004)

Studied pushover methods

For investigating the efficiency and accuracy of energy-based Capacity spectrum method, the structural models are subjected to the different nonlinear static procedures (NSP) analysis. The selected methods are in two conventional and advanced procedure groups. The conventional procedures are based on the first mode (M1), triangular and uniform load patterns. The advanced methods are divided into two sections, i.e., MPA procedure as a multi-run procedure and the SSP procedure as a single-run procedure that the basic concepts and method of determining the applied load pattern of them will be described in the introduction section.

The number of vibration modes in the modal pushover analysis methods regarding FEMA 273 is chosen such that the accumulation of the modal participation mass is over 90% (FEMA, 1997). For this purpose, the first three and four elastic modes respectively are used for SAC-9 and SAC-20 buildings in MPA and SSP methods.

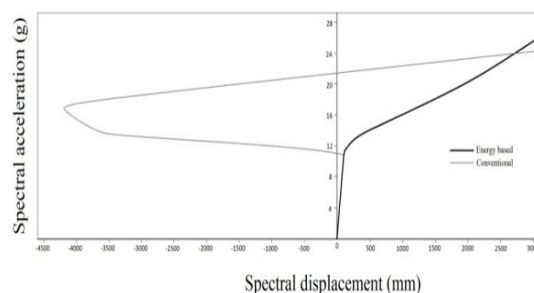


Figure 2. Capacity curve obtained in third-mode pushover analysis, plotted using current MPA method and using the energy-based displacement method.

Determination of target displacement

In pushover analysis methods, the target displacement is approximated through the maximum inelastic displacement of the equivalent SDOF system. In this paper, for determining the target displacement, capacity spectrum curve (F^*-D curve) of the equivalent SDOF system with unit mass is established in two different conventional roof-displacement approach and energy-based approach. The process of converting a

MDOF system into an equivalent SDOF system in establishment of capacity curve based conventional roof-displacement approach and energy-based approach will be described in detail in previous section. The derived capacity curve for the equivalent SDOF system is idealized as a bilinear curve according to the existing process in FEMA 356 and the peak displacement of the bilinear inelastic SDOF system (target displacement) is computed through the NTH analysis.

Inter-story drift and story displacement prediction

In order to investigate the accuracy and efficiency of the capacity spectrum method based on energy approach, the inter-story drifts as important indicator of vulnerability in damages of structures and the story displacement are evaluated.

In each investigated pushover analysis methods, the responses resulting from energy-based capacity spectrum method and conventional capacity spectrum method (based on roof-displacement approach), are compared together and to the responses of the NTH analysis as a benchmark responses.

To compare the responses, the total error index in the estimation of the inter-story drift and displacement for each pushover analysis methods respectively is defined by equations (7) and (8) (Lopez-Menjivar and Pinho, 2004). And the relative error index in the estimation of the inter-story drift and displacement for each story respectively is defined by equations (9) and (10).

$$Error_{\Delta}(\%) = 100 \times \frac{1}{n} \sqrt{\sum_{i=1}^n \left(\frac{\Delta_{i-NTHA} - \Delta_{i-Push}}{\Delta_{i-NTHA}} \right)^2} \quad (7)$$

$$Error_{Dis}(\%) = 100 \times \frac{1}{n} \sqrt{\sum_{i=1}^n \left(\frac{Dis_{i-NTHA} - Dis_{i-Push}}{Dis_{i-NTHA}} \right)^2} \quad (8)$$

$$Error_{\Delta_i}(\%) = 100 \times \left(\frac{\Delta_{i-Push} - \Delta_{i-NTHA}}{\Delta_{i-NTHA}} \right) \quad (9)$$

$$Error_{Dis_i}(\%) = 100 \times \left(\frac{Dis_{i-Push} - Dis_{i-NTHA}}{Dis_{i-NTHA}} \right) \quad (10)$$

where, Δ_{i-NTHA} is the peak inter-story drift at a given level i , resulting from the NTH analysis, Δ_{i-Push} is the corresponding inter-story drift of the NSP, Dis_{i-NTHA} is the peak story displacement at a given level i , resulting from NTH analysis, Dis_{i-Push} is the corresponding displacement of the NSP and n is the number of the stories. In this paper, the average responses of 10 records for each pushover analysis methods for SAC9 and SAC20 buildings are shown in Figures 3 up to 6.

As presented in Figures 3 up to 6, in each pushover analysis method, except uniform load pattern, the error of responses resulting from energy-based capacity spectrum is less than those associated with the conventional capacity spectrum method (based on roof-displacement approach). As can be observed in Figures 3 and 5, applying M1 or Triangular load patterns in energy-based capacity spectrum procedure and in roof-displacement procedure does not lead to a significant distinction in estimating the inter-story drifts. While, in the MPA and SSP methods the responses resulting from energy-based capacity spectrum show an admirable accuracy in comparison with the responses resulting from roof-displacement approach. The reason can be inferred from this fact that in the M1 and Triangular methods, changes of the roof displacement largely are proportional to the changes of other stories displacement due to the type of lateral load pattern. Consequently, in the M1 and Triangular methods, generated capacity spectrum based on energy approach is same to the conventional capacity spectrum. But, in the MPA and SSP methods where the effects of higher modes are participated in analysis, changes of the roof displacement is not proportional to the changes of other stories displacement due to the type of lateral load pattern. Hence, in the MPA and SSP methods the generated capacity spectrum based on energy approach is different with conventional capacity spectrum. As presented in Figures 4 and 6, the accuracy of energy-based spectrum procedure in estimating the stories displacement is noticeably higher than the roof-displacement method.

Table1. Ground motions properties.

Earthquake	Year	Recording Station	Component	PGA(g)	PGV(Cm/s)	Closest to fault rupture(km)	Mag.
Northridge	1994	77 Rinaldi	228	0.838	166.1	7.1	6.7
Loma perieta	1989	16 LGPC	0	0.563	94.8	6.1	6.9
Landers	1992	24 Lucerne	275	0.721	97.6	1.1	7.3
Cape Mendocino	1992	89156 Petrolia	90	0.662	89.7	9.5	7.1
Kobe	1995	0 KJMA	0	0.821	81.3	0.6	6.9
Erzican	1992	95 Erzican	NS	0.515	83.9	2	6.9
Duzce	1999	Duzce	270	0.535	83.5	8.2	7.1
Gazli	1976	9201 Karakyr	90	0.718	71.6	--	6.8
Chi-Chi	1992	CHY028	W	0.653	72.8	7.3	7.6
Tabas	1978	9101 Tabas	LN	0.836	97.8	--	7.4

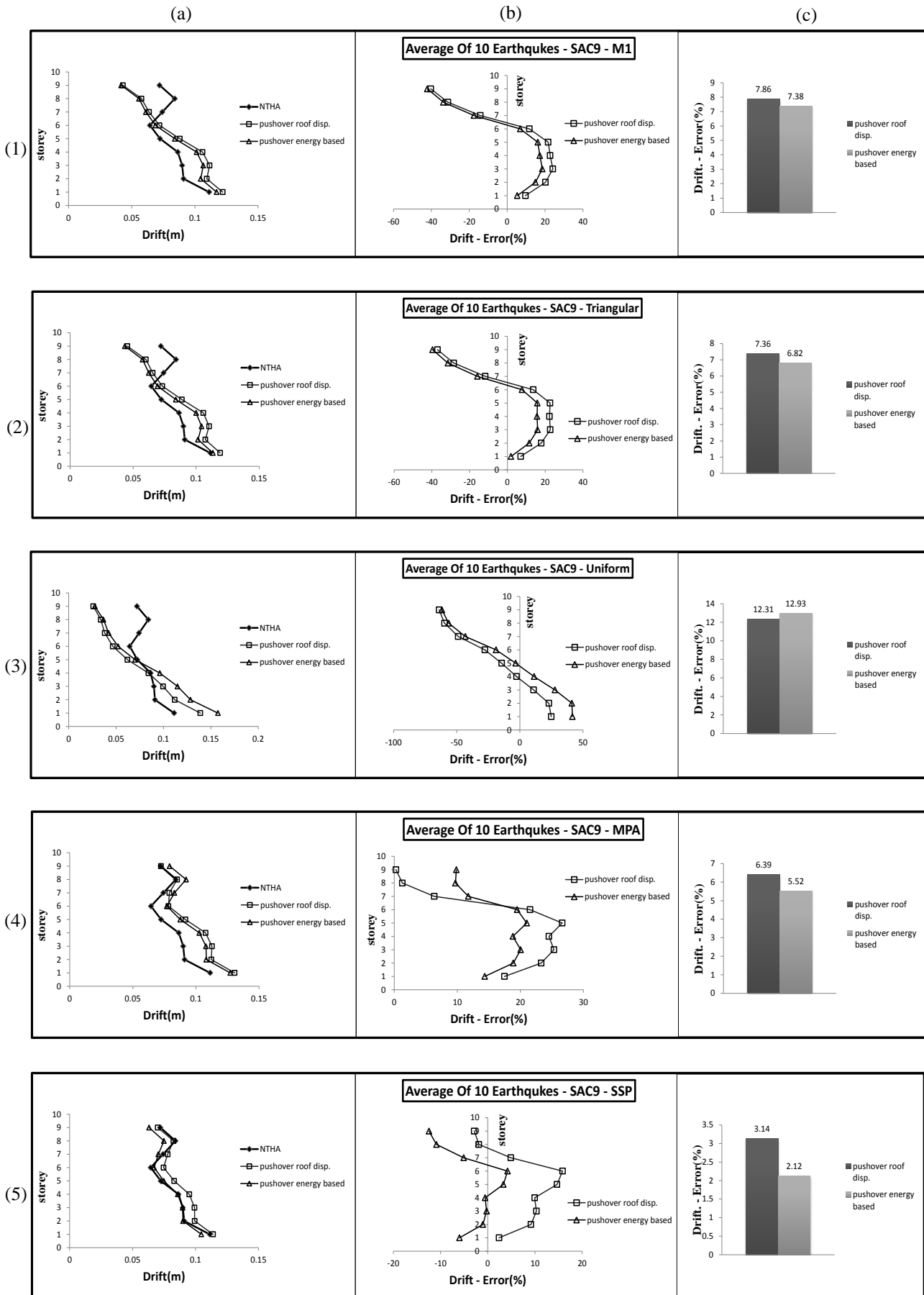


Figure 3. Average of responses profiles resulting from 10 earthquakes and the different NSPs, 1) First mode (M1), 2) Triangular, 3) Uniform, 4) MPA, 5) SSP, based on energy approach and roof-displacement approach, for the SAC9 building. (a) Inter-story drift, (b) Error of the inter-story drift in the different story levels, (c) Total error of the inter-story drift in all story levels.

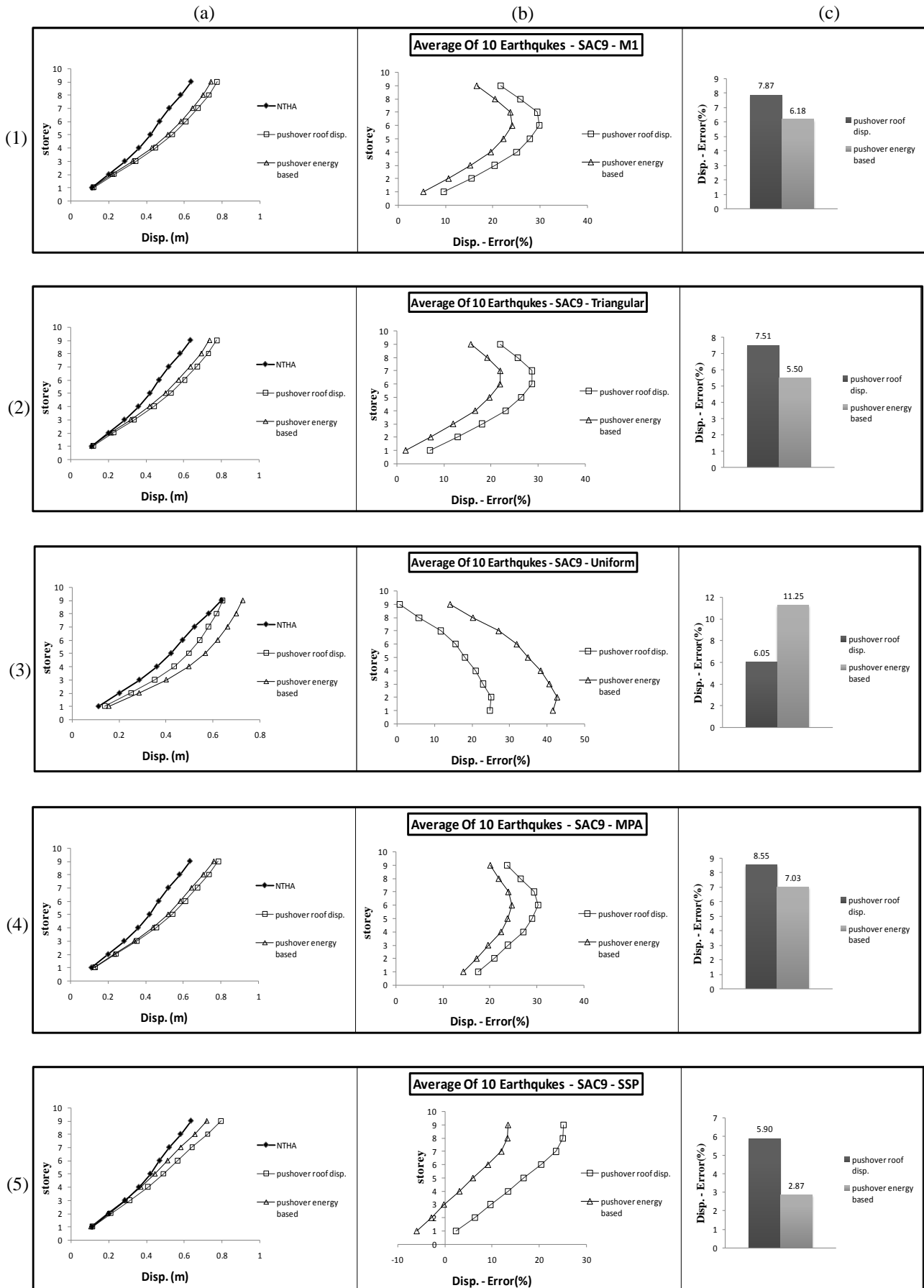


Figure 4. Average of responses profiles resulting from 10 earthquakes and the different NSPs, 1) First mode (M1), 2) Triangular, 3) Uniform, 4) MPA, 5) SSP, based on energy approach and roof-displacement approach, for the SAC9 building. (a) story displacement, (b) Error of the story displacement in the different story levels, (c) Total error of the story displacement in all story levels.

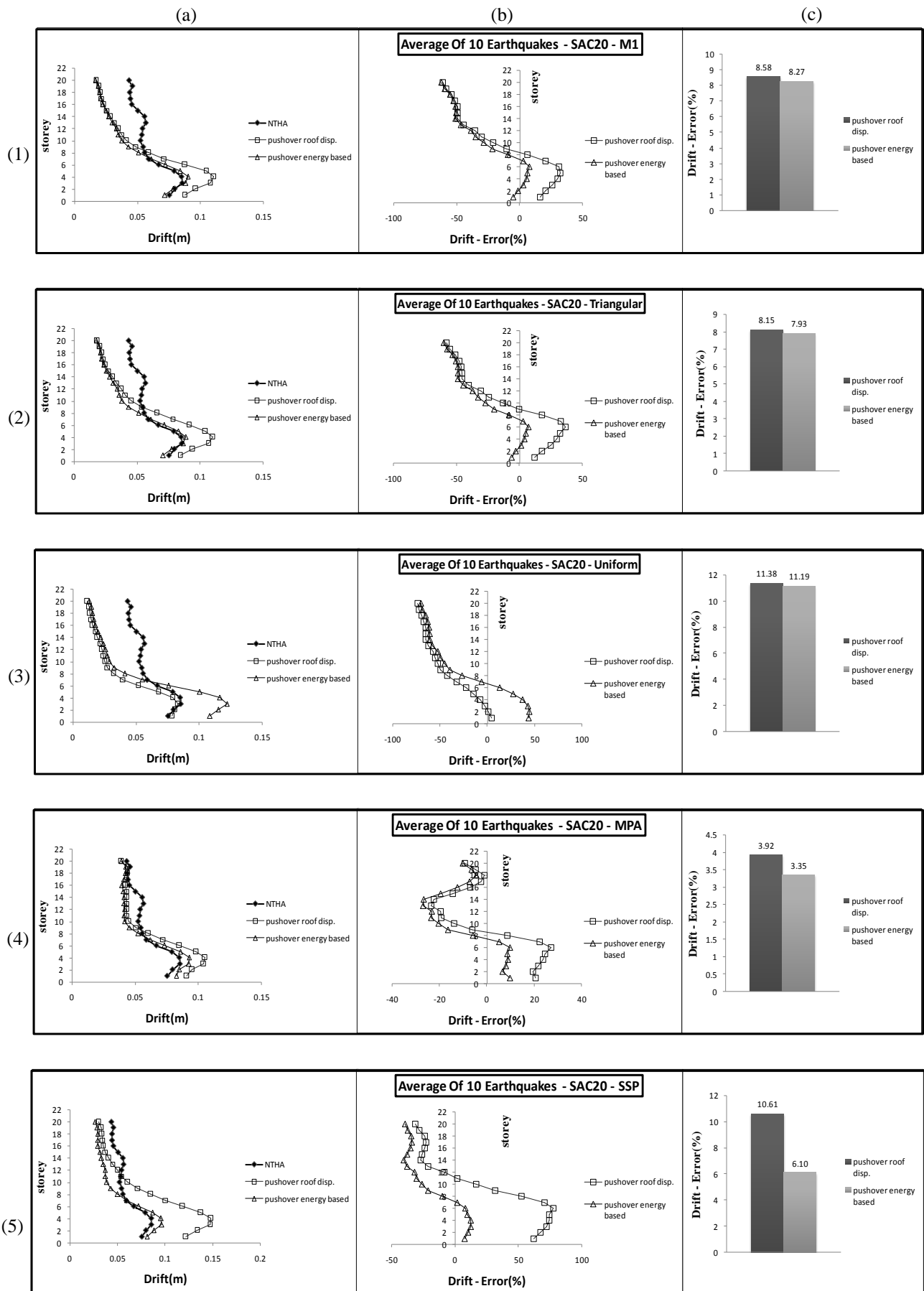


Figure 5. Average of responses profiles resulting from 10 earthquakes and the different NSPs, 1) First mode (M1), 2) Triangular, 3) Uniform, 4) MPA, 5) SSP, based on energy approach and roof-displacement approach, for the SAC20 building. (a) Inter-story drift, (b) Error of the inter-story drift in the different story levels, (c) Total error of the inter-story drift in all story levels.

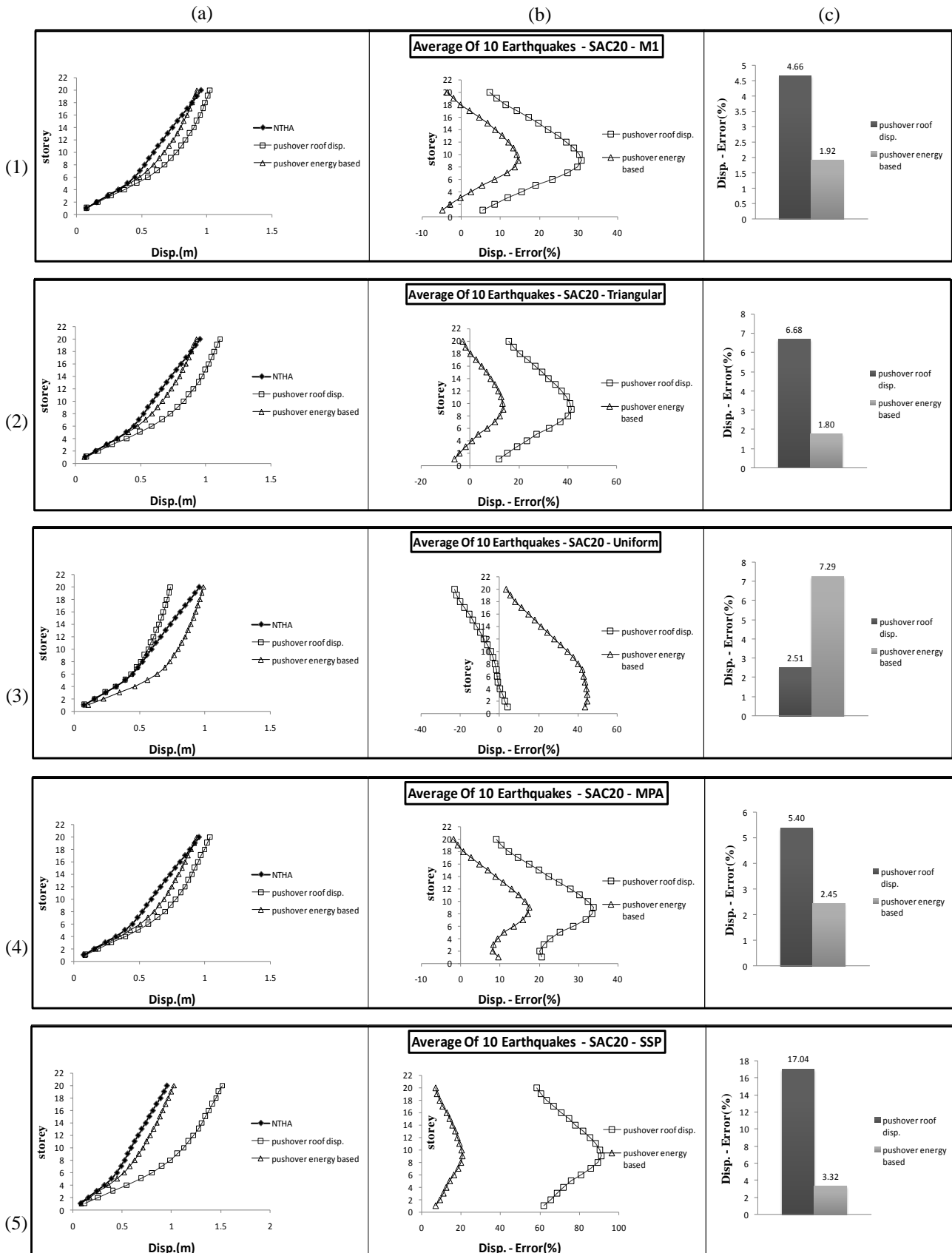


Figure 6. Average of responses profiles resulting from 10 earthquakes and the different NSPs, 1) First mode (M1), 2) Triangular, 3) Uniform, 4) MPA, 5) SSP, based on energy approach and roof-displacement approach, for the SAC20 building. (a) story displacement, (b) Error of the story displacement in the different story levels, (c) Total error of the story displacement in all story levels.

SUMMARY AND CONCLUSION

Since in the higher modes the increase of the roof displacement is not proportional to the other stories

displacement and even may reverse, using the roof displacement for establishing the capacity spectrum in the higher modes is faced with ambiguities. Using the energy

concepts eliminates the existent ambiguities and problems over choosing roof as control point of displacement in pushover procedures. Also, comparing the results of various investigated pushover analysis methods with those resulting from NTH analysis in this study shows that using the energy-based capacity spectrum can estimate the responses more accurately in comparison with using conventional capacity spectrum. Consequently, it can be stated that using energy-based capacity spectrum is preferable to using conventional capacity spectrum (based on roof-displacement approach) both in terms of efficiency and accuracy in estimating the responses of NTH analysis.

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