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The Effect of Post-Yield Stiffness Ratio on Strength Reduction Factors Due to Ductility with an Emphasis on Iran's Strong Ground Motions

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ORIGINAL ARTICLE

ABSTRACT: Inelastic strength demands can be calculated from elastic strength demands by strength reduction factors. These factors are also called response modification factors which depend on some parameters such as ductility, over-strength, redundancy and damping. The main objective of this study is to determine the strength reduction factor due to ductility for non-deteriorating bilinear single degree of freedom systems which have positive post-yield stiffness or strain hardening for some Iran's major earthquake records. Compact structural steel sections and some seismic base isolation systems are some examples of mechanical systems with bilinear hysteresis behaviour. In this research, The effect of positive post-yield stiffness ratio is studied for both constant-ductility response spectra or inelastic strength demand spectra and strength reduction factors due to ductility. The evaluation of ductility factor requires large computational efforts. To overcome this problem, the PRISM software was used. A functional form was proposed and parameters were evaluated by the least square error method. This functional form is assumed to be a function of post-yield stiffness ratio, target ductility factor and structural period. For this purpose, the El-Centro, Tabas, Naghan and Abbar ground motions were used.

Keywords: Inelastic Strength Demand, Strength Reduction Factor, Target Ductility Factor, Bilinear Hysteresis Model

INTRODUCTION

The main purpose of seismic design is to be sure that the strength and deformation capacities of structures will not exceed the demands which are imposed by severe earthquakes, with an adequate safety margin. The earthquake forces - which are applied on structural systems with elastic behaviour - under strong base excitations are commonly very large and taking into account such large forces for designing of structures are not economically reasonable. Most seismic design codes allow structures to act inelastically during strong earthquake base excitations. To evaluate inelastic strength demand spectra and strength reduction factors for nonlinear single degree of freedom (SDOF) systems, hysteresis loops can be used to define the nonlinear behaviour of structural members and materials. The effect of various hysteresis models and parameters on ductility and strength spectra had been studies by some researchers. Newmark and Hall [1] proposed a functional form to evaluate the strength reduction factor due to ductility for elastic-perfectly plastic SDOF systems with 5% of damping and for three ground motion records. The evaluating of the inelastic response spectra from the elastic response spectra can be performed with their proposed functional form. Nassar and Krawinkler [2] developed a functional form to evaluate response reduction factors with respect to ductility for firm soil sites in the western United States. They used 15 ground motion records and their functional form was considered the ductility, natural period and post-yield stiffness slope of a bilinear model. Miranda and Bertero [3] used 124 earthquake records to evaluate response reduction factors due to ductility for different soil types.

In this study, a different approach was used to obtain a functional form to evaluate the response reduction factors with respect to ductility with the help of inelastic strength demand spectra. For this purpose, the non-deteriorating bilinear hysteretic model with initial stiffness of k, hardening stiffness of αk and yield strength of F_y were used to consider the inelastic behaviour of an SDOF system. Base isolation systems can also be characterized by a bilinear hysteretic model. Inelastic strength demand spectra were evaluated for a range of positive post-yield stiffness ratios for four ground motion records. In all analyses 5% of damping was assumed. The Newmark-β numerical integration with constant average acceleration coefficients was used to solve nonlinear SDOF motion equation.

In order to accomplish this research, Excel, PRISM and TableCurve 3D softwares were used.

EARTHQUAKE RECORDS

A set of four strong motion records were used to evaluate strength demands and strength reduction

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factors. All records were scaled to a constant PGA of 0.35g to conform to the zone 1 of the Iranian standard Code 2800 [7]. Three Iran's earthquake ground motions were chosen with an additional 1940's El-Centro ground motion to be comparable with the results of other researchers. Table 1 shows the general details of ground motions used in this study.

Table 1. Ground motion records used in this study

Name	Duration (s)	PGA before scaling (g)	PGA after scaling (g)
Tabas (1978)	25.02	0.934	0.35
Naghan (1977)	5.00	0.724	0.35
Abbar (1990)	53.50	0.515	0.35
El-Centro (1940)	31.18	0.319	0.35

Figures 1 to 4 show the ground accelerations for selected earthquakes and Figure 5 shows the elastic strength demand spectra for an SDOF system with 5% of critical damping for four scaled ground motion accelerations plus mean spectrum.

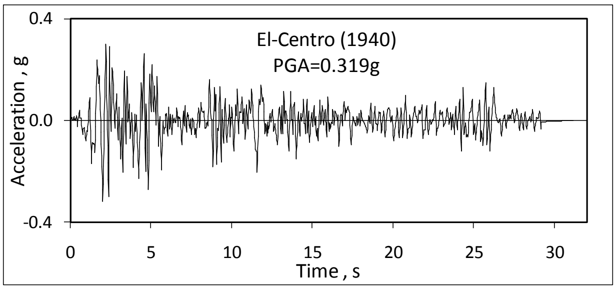

Figure 1. El-Centro ground acceleration

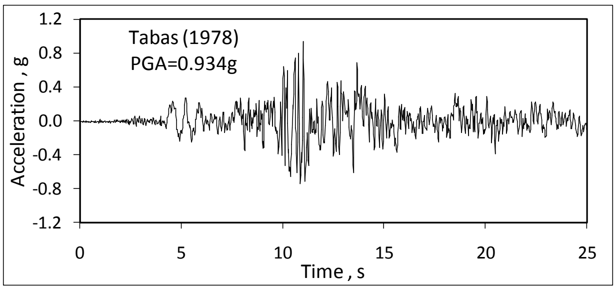

Figure 2. Tabas ground acceleration

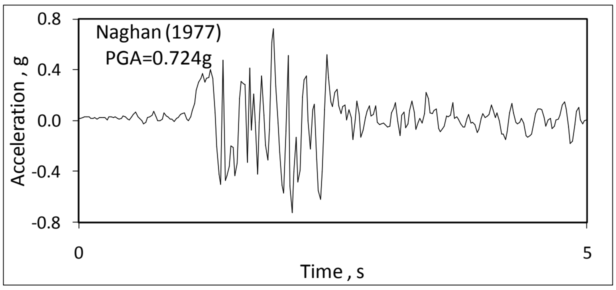

Figure 3. Naghan ground acceleration

Nonlinear time history analysis

The equation of motion of a nonlinear SDOF system is also mathematically nonlinear (Equation 1).

$$m\ddot{u} + c\dot{u} + F_s(u) = -m\ddot{u}_g \qquad (1)$$

Where m is the mass of SDOF system, c is the viscous damping coefficient, u is the relative displacement between mass and ground, $\ddot{u}_g$ is the ground acceleration and $F_s(u)$ is the restoring force. Bilinear hysteresis model was used to define nonlinear behaviour of an SDOF system (Figure 6). It is generally used to define the behaviour of structural elements which has inadequate strength or stiffness degradation, for example flexural of a compact steel beam which lateral or torsional buckling is not important [2]. The most practical method for step by step integration of the equation of motion is Newmark-β method which had been used in this study.

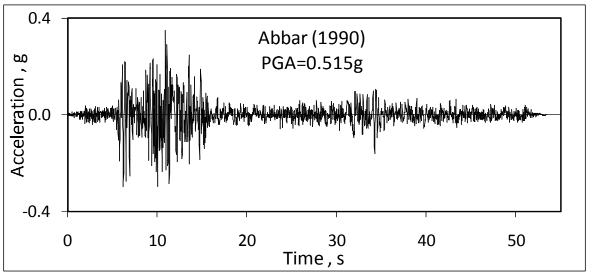

Figure 4. Abbar ground acceleration

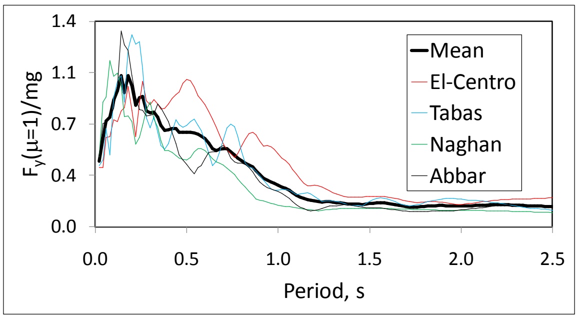

Figure 5. Scaled elastic response spectra of a bilinear SDOF system plus mean spectrum

Nonlinear time history analyses on SDOF systems were performed using PRISM, a computer program for nonlinear seismic response analysis of SDOF system [4]. The PRISM software uses Newmark-β method to solve for elastic and inelastic spectra. Constant average acceleration coefficients were used to have a high degree of numerical stability [5].

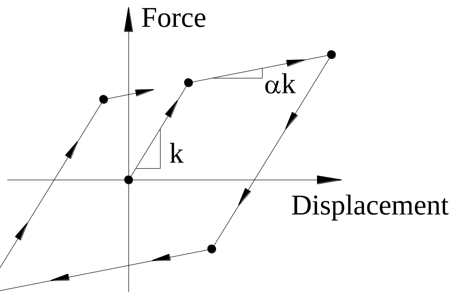

Figure 6. Bilinear hysteretic model used in this study

The nonlinear time history analyses were performed for the following 480 permutations:

- For a set of 4 earthquake records.

- For target ductility factors, μ=1, 1.5, 2, 3, 4, 5, 6 and 8.
- For post-yield stiffness ratios, α=0, 1, 2, 3, 4, 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50.

All results are given for 5% critical damping. The critical damping was not varied as a parameter in this study.

Evaluation of inelastic strength demand spectra

Strength demand is defined as $F_y(\mu)/mg$, where equals to yield level F_y over seismically effective weight mg for a target ductility factor μ. The elastic strength demand spectrum is equal to the acceleration response spectrum where inelastic strength demand spectrum illustrates the period dependant yield level required to limit the ductility factor to a prescribed value of ductility factor [2]. Figures 7 to 10 show the mean inelastic strength demand spectra for a bilinear SDOF system with post-yield or strain hardening stiffness of 0, 2, 10 and 50 percents of initial stiffness. Other post-yield stiffness ratios were not illustrated and were used for regression purposes.

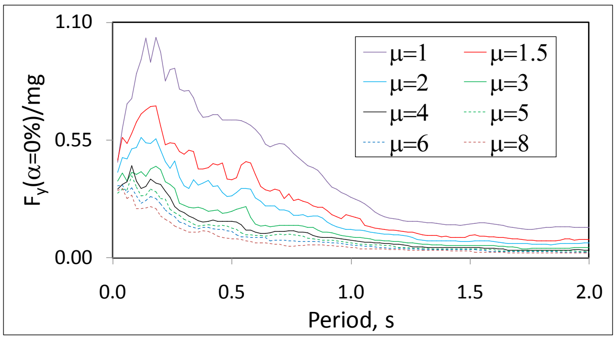

Figure 7. Mean inelastic strength demand spectra for bilinear SDOF system and α=0%

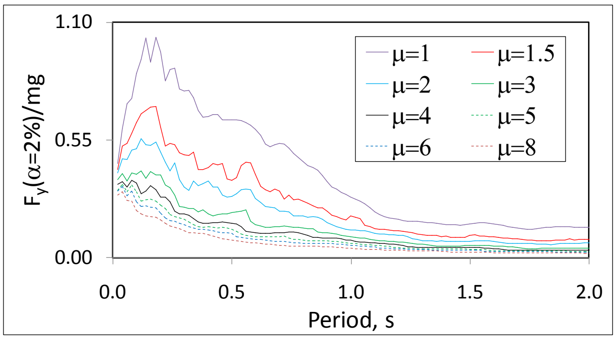

Figure 8. Mean inelastic strength demand spectra for bilinear SDOF system and α=2%

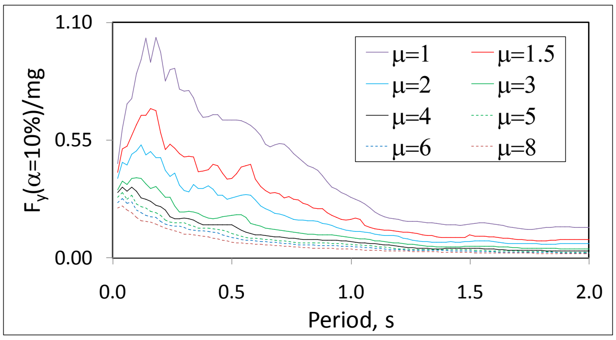

Figure 9. Mean inelastic strength demand spectra for bilinear SDOF system and α=10%

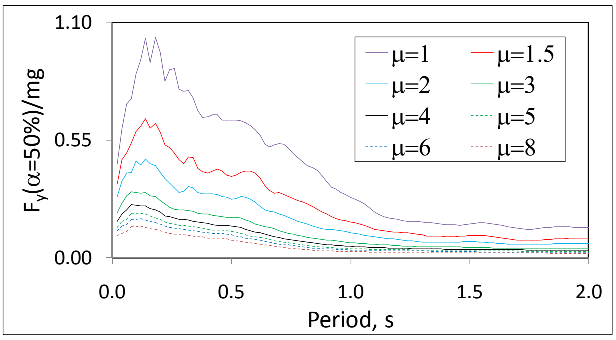

Figure 10. Mean inelastic strength demand spectra for bilinear SDOF system and α=50%

Figures 7 to 10 where plotted using PRISM software. By investigating the 3D graphs of the relationship between inelastic strength demand, post-yield stiffness ratio α and target ductility factor μ (Figures 11 to 13), a parametric functional form for regression was proposed (by using TableCurve 3D software) as follows. The regression procedure is given in the following section.

$$\left(\frac{F_y(\mu)}{m\cdot g}\right)_T = a + b\cdot\alpha + \frac{c}{\mu} + d\cdot\alpha^2 + \frac{e}{\mu^2} + f\cdot\frac{\alpha}{\mu} \qquad (2)$$

The subscript, T indicates the period where the unknown coefficients of a to f were estimated for the equation. Also, α is the post-yield stiffness ratio and μ is target ductility factor which is defined as the maximum deformation to the yield deformation for a system with yield strength smaller than the elastic strength demand.

$$\mu = \frac{u_m}{u_y} \qquad (3)$$

u_m is the maximum absolute deformation of a nonlinear SDOF system due to the ground motion. u_y is the deformation at which yielding begins. $F_y(\mu)$ is the inelastic strength demand which defines the yield strength required of an inelastic system in order to limit the ductility demand to a target value of μ. The least square error method was employed in the regression analyses.

The way of choosing equation (2) by tablecurve 3d

The basic polynomials inside TableCurve 3D consist of 243 basic equations based upon *X*, *ln(X)*, *1/X*, *Y*, *ln(Y)*, and *1/Y*. These consist of 225 non-interactive equations and 18 Taylor polynomials which include X-Y interaction terms.

TableCurve 3D's Selective Subset algorithm seeks to select from all possible combinations of X-basis functions and Y-basis functions, those equations having up to nine coefficients which produce the best least-squares fits.

The Selective Subset algorithm must necessarily do all three-parameter fits first, then the four-parameter, and so on through to the nine parameter fits. In each level, a merit function is assigned to each basis function. Those least contributing to the various fits are discarded before the next level begins.

In defence of “Best Subset” and “All Possible Subset” algorithms, it is important to note that their purpose is primarily not to deal with multiple basis functions of one or two variables, but to deal with a large number of independent variables. In such instances, it is quite appropriate to fit every possible permutation to insure each independent variable’s impact is fully appraised.

Non-linear equations are those which cannot be solved in a single step solution of a matrix, but must be managed in an iterative fashion. As such non-linear fitting is a much slower process and one that often requires starting estimates in order to initiate the fitting.

TableCurve 3D’s non-linear fitting incorporates Levenburg-Marquardt algorithm that uses the Gauss-Jordan procedure for the matrix inverse required in each iteration. In non-linear fitting, the parameters are iteratively adjusted to minimize a goodness of fit merit function. If the algorithm is fully successful, a true global minimum (the true least-squares fit) is achieved.

Since the Levenburg-Marquardt method’s minimization procedure requires the partial derivatives with respect to the parameters, TableCurve 3D uses analytic derivatives for the 168 built-in functions for the highest precision and the most rapid fitting and convergence.

The Levenburg-Marquardt algorithm requires starting estimates for the adjustable parameters. For the 168 built-in non-linear equations, these are automatically supplied by the pre-scan procedure.

The non-linear fitting algorithm can be configured for the maximum number of iterations as well as to specify a convergence criterion. TableCurve 3D deems the algorithm converged when the r^2 coefficient of determination is unchanging in the significant digit specified for five consecutive iterations. The fitting engine can also be interrupted to terminate a non-linear fit that is going nowhere, but is still slowly changing in the r^2 merit function.

Sum of Squares due to Error (Sum of Residuals Squared):

$$SSE = \sum_{i=1}^{n} \omega_i \left(\hat{z}_i - z_i\right)^2 \quad (4)$$

Sum of Squares about Mean:

$$SSM = \sum_{i=1}^{n} \omega_i \left(z_i - \bar{z}\right)^2 \quad (5)$$

Coefficient of Determination:

$$r^2 = 1 - \frac{SSE}{SSM} \quad (6)$$

Where r^2 is close to unit, the error is negligible. TableCurve 3D gave several equations in which r^2 was close to unit. From those equations, the equation which had least parameters was chosen. It was the simplest one that could be used (for example equation (2)) [8].

RESULTS

Table 2 shows the regression function coefficients for periods of 0.1, 0.2, 0.5, 1, 1.5 and 2 seconds which the period of most buildings lays in this range. Values in this table where obtained by using TableCurve 3D software.

Table 2. Parameters of the new functional form for strength reduction factor due to ductility

Period (s)	a	b	c	d	e	f
0.1	0.1437	-0.3705	0.7785	0.2714	-0.0529	0.2333
0.2	0.1318	-0.4793	0.5572	0.5720	0.2925	0.1648
0.5	0.0520	-0.1264	0.3394	0.1919	0.2513	0.0453
1.0	0.0265	-0.0865	0.1893	0.0488	0.0717	0.0485
1.5	0.0139	-0.0294	0.1182	0.0229	0.0268	0.0191
2.0	0.0067	-0.0188	0.1028	0.0341	0.0315	0.0071

Figures 11 to 13 show the 3D perspective view of the points which are extracted from the inelastic strength demand spectra and regressed surface for periods 0.1, 0.5 and 2 seconds. These figures were plotted by using TableCurve 3D software.

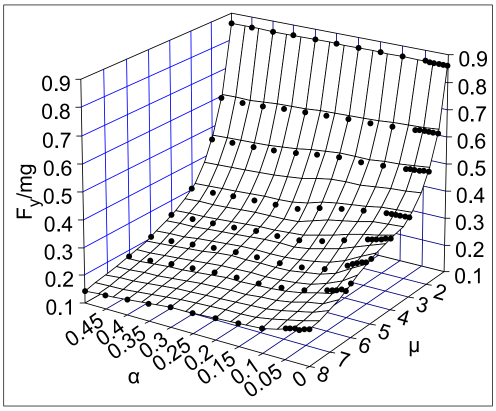

Figure 11. A 3D perspective view of the extracted points and the regressed surface at period of 0.1 seconds

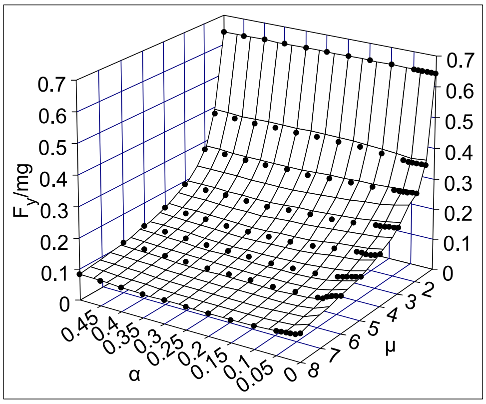

Figure 12. A 3D perspective view of the extracted points and the regressed surface at period of 0.5 seconds

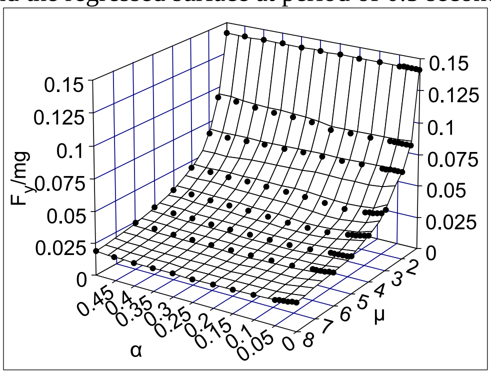

Figure 13. A 3D perspective view of the extracted points and the regressed surface at period of 2.0 seconds

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Evaluation of strength reduction factor due to ductility

The strength reduction factor due to ductility or $R_y(\mu)$ is defined as the ratio of elastic strength demand $F_y(\mu=1)$ to the inelastic strength demand $F_y(\mu=\mu_i)$ for a nonlinear SDOF system,

$$R_y(\mu)=\frac{F_y(\mu=1)}{F_y(\mu=\mu_i)} \quad (7)$$

The relationship between $F_y(\mu=1)$ and $F_y(\mu=\mu_i)$ is shown in Figure 14 [6].

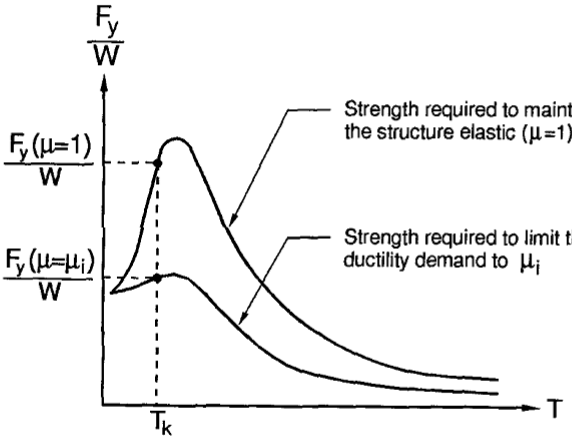

Figure 14. Linear and constant ductility nonlinear response spectra

After replacing $F_y(\mu)$ (Equation 2) into the equation (7) and simplification, strength reduction factor due to ductility can be expressed as follows,

$$(R_y(\mu))_T=\frac{(a+c+e+\alpha(b+f+d\cdot\alpha))\mu^2}{e+\mu(c+f\cdot\alpha+(a+\alpha(b+d\cdot\alpha))\mu)} \quad (8)$$

The subscript, T indicates the period where the unknown coefficients of a to f were estimated for the equation. Also, α is the post-yield stiffness ratio and μ is target ductility factor. Figures 15 and 16 show the regression values of strength reduction factor due to ductility $R_y(\mu)$ for a bilinear SDOF system with positive post-yield stiffness ratios of α=0 and 10%.

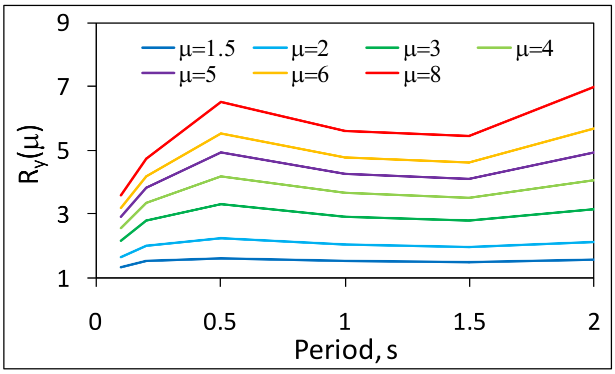

Figure 15. Regression values of response reduction factor due to ductility for bilinear SDOF system with α=0%

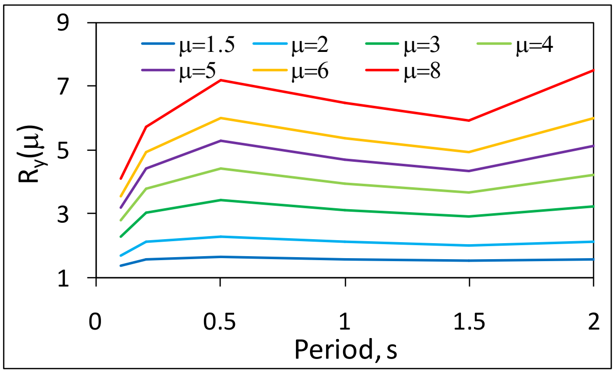

Figure 16. Regression values of response reduction factor due to ductility for bilinear SDOF system with α=10%

In figures 17, 18 and 19, the results were compared with Nassar and Krawinkler's work [2] which are summarized in the following formula. They proposed a functional form and evaluated the parameters for post-yield stiffness ratios of α=0, 2 and 10%. To be more readable, the graphs were drawn only for target ductility factors of μ=2, 4, 6 and 8.

$$R_\mu=[c(\mu-1)+1]^{1/c} \quad (9)$$

Where

$$c(T,\alpha)=\frac{T^a}{1+T^a}+\frac{b}{T} \quad (10)$$

α is the post-yield stiffness as percentage of initial stiffness of the bilinear SDOF system and parameters a and b are given in Table 3.

Table 3. Parameters for Nassar and Krawinkler's proposed functional form

α (%)	a	b
0.00	1.00	0.42
0.02	1.00	0.37
0.10	0.80	0.29

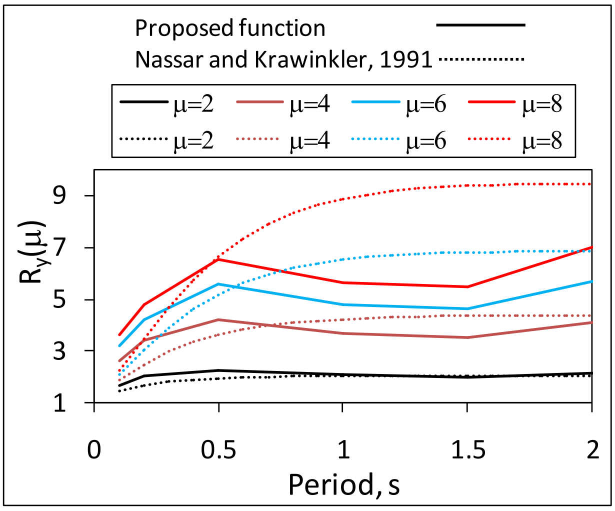

Figure 17. Comparison of proposed functional form for response reduction factor due to ductility for bilinear SDOF system with α=0% and Nassar and Krawinkler [2]

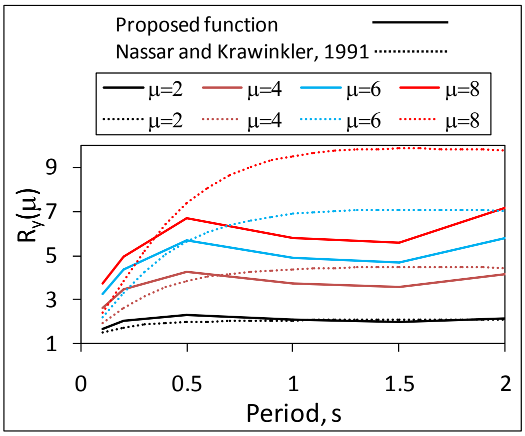

Figure 18. Comparison of proposed functional form for response reduction factor for bilinear SDOF system with α=2% and Nassar and Krawinkler [2]

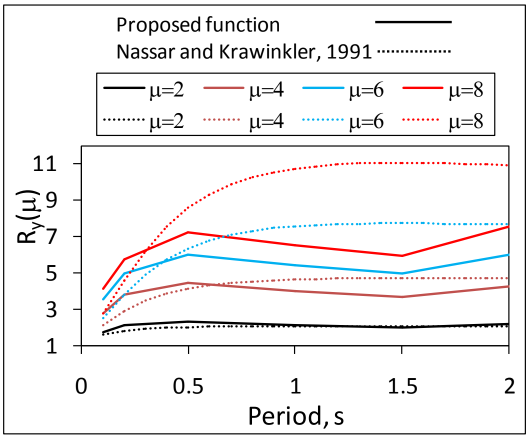

Figure 19. Comparison of proposed functional form for response reduction factor for bilinear SDOF system with α=10% and Nassar and Krawinkler [2]

For periods less than 0.5 seconds, there is a good agreement between the present proposed functional form and Nassar and Krawinkler's work but for periods greater than 0.5 seconds, there are more discrepancy when the ductility factors get a value greater than 5. For example, as can be seen in Figure 17, for an elastic-perfectly plastic SDOF system with target ductility factor of 4, the strength reduction factor due to ductility according to this paper's proposed function at period of 1 second is 3.67 and according to Nassar and Krawinkler's formula [2] is 4.22 but for same period, at target ductility factor of 8, this paper's proposed function is equal to 5.61 but Nassar and Krawinkler's formula is equal to 8.86. To investigate the reason why there are such discrepancies between the results of this paper's proposed function and Nassar and Krawinkler's proposed function, the graphs of strength reduction factor due to ductility versus period was drawn for four earthquake ground motions mentioned in this paper and for different target ductility factors. Figures 20 to 23 shows the strength reduction factor for target ductility factors of μ=2, 4, 6 and 8 and bilinear elastic-perfectly plastic SDOF system for El-Centro, Tabas, Naghan and Abbar ground accelerations. It can be observed that for target ductility factors greater than 4, the strength reduction factors due to ductility have large discrepancies in different ranges of periods.

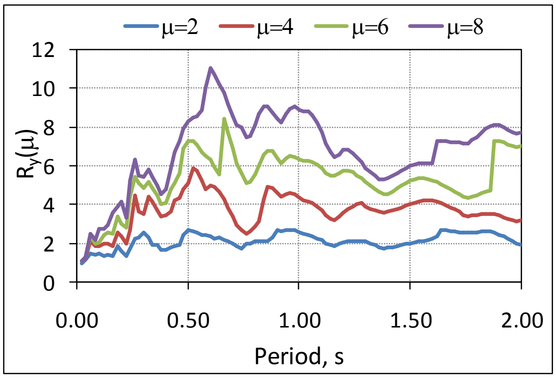

Figure 20. Strength reduction factor due to ductility for bilinear SDOF system with α=0% for El-Centro earthquake

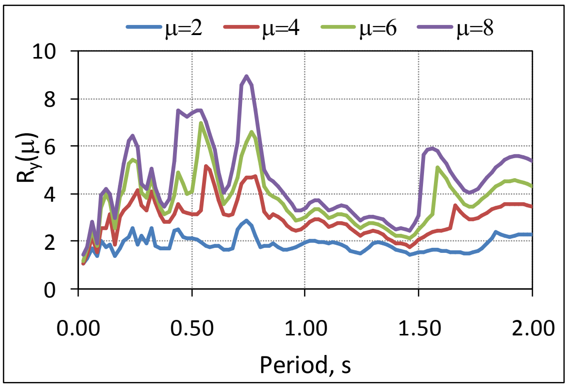

Figure 21. Strength reduction factor due to ductility for bilinear SDOF system with α=0% for Tabas earthquake

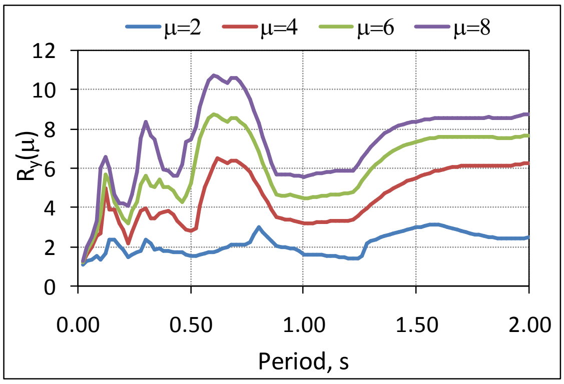

Figure 22. Strength reduction factor due to ductility for bilinear SDOF system with α=0% for Naghan earthquake

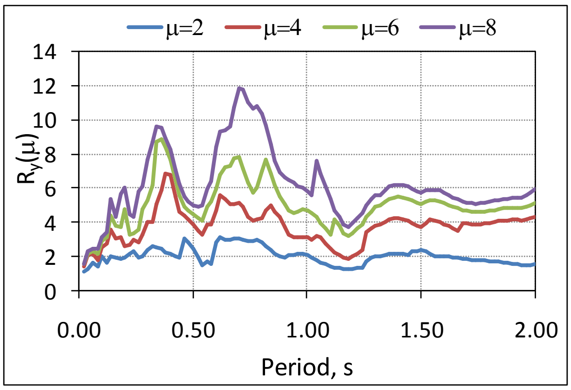

Figure 23. Strength reduction factor due to ductility for bilinear SDOF system with α=0% for Abbar earthquake

Figures 15 to 23 were plotted using PRISM and Excel softwares.

CONCLUSION

The most important objective of this research is to study the influence of positive post-yield stiffness on strength reduction factors due to ductility on an SDOF system with bilinear hysteresis model for several Iran's strong ground motions. To attain this objective, for ordinary building periods range, a functional form which retained simplicity and minimum number of parameters was proposed for regression of inelastic strength demand spectra which could be used to evaluate strength reduction factors due to ductility. This paper presented parameters that can be used to evaluate the response reduction factors due to ductility with respect to period, target ductility factor and post-yield stiffness ratio. The parameters of the proposed functional form were calculated by least square error method.

The most well-known formula for strength reduction factor due to ductility that incorporates the post-yield stiffness ratio had been developed by Nassar and Krawinkler [2]. The curve obtained by them was computed with several ground motions recorded in firm sites in the United States. They recommended a simplified expression for strength reduction factor due to ductility. Their proposed expression had some discrepancy when the periods were greater than 0.5 seconds and ductility factors were greater than 5. The reason can be realized from the graphs of strength reduction factor - due to ductility versus period - obtained from four Iran’s major earthquake ground motions mentioned in this paper and for different target ductility factors. In general, the results from our proposed discrete functional form has more agreement with exact values of the strength reduction factor due to ductility for Iran’s major earthquake ground motions mentioned in this paper.

The following general conclusions can be drawn from this study:

1. Independently of the post-yield stiffness ratio, the strength reduction factor due to the ductility must be significantly verified when the natural period of the structure is greater than 0.5 seconds.
2. This study suggests that the effect of post-yield stiffness ratio must be considered to evaluate the strength reduction factors due to ductility.

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