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A Comparison between Performances of Turbulence Models on Simulation of Solitary Wave Breaking by WCSPH Method

Amin Mahmoudi*[1], Habib Hakim zadeh[2], Mohammad Javad Ketabdari[3]

[1,2]*Faculty of Civil Engineering, Sahand University of Technology, Tabriz, Iran*
[3] *Faculty of Marine Technology, Amirkabir University of Technology, Tehran, Iran*

*Corresponding author's Email: a_mahmoodi@sut.ac.ir

ABSTRACT: The modeling of Solitary wave breaking is an important subject in coastal and marine engineering, because the damage associated with tsunamis is related to their wave breaking and run-up on the shoreline. In this paper a space-averaged Navier–Stokes approach has been deployed to simulate the Solitary wave breaking on a plane slope. The developed model is based on the smoothed particle hydrodynamic (SPH) method which is a pure Lagrangian approach and can handle large deformations of the free surface with high accuracy. Since breaking waves are characterized by high distortion, the turbulence modeling has been of major concern in such simulations. For this study, the large eddy simulation (LES) model was chosen as the turbulence model to couple with the weakly compressible version of the smoothed particle hydrodynamics (WCSPH) method to simulate the Solitary wave breaking on a plane slope. In order to investigate the Effects of turbulence models on numerical simulations of wave breaking, the results of present study were compared with experimental results and numerical data found in the literatures. The results shown that turbulence modeling have a strong influence on the quality of the results. Furthermore, capability of the WCSPH method along with a LES approach to simulate the Solitary wave breaking on a plane slope was compared with result of ISPH method in the literatures. Finally, it is shown that the WCSPH coupled with LES model performs better than ISPH method.

Keywords: WCSPH, solitary wave, Turbulence model, LES.

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INTRODUCTION

Tsunamis are ocean waves that have long wavelengths and small wave heights. They have been generated by several geophysical events such as: earthquakes, landslides, volcano eruptions, and other mechanisms such as underwater explosions. Thus, the energy associated with a tsunami can be very large and it can destroy everything located at coast. As tsunami waves propagate shoreward, the wave heights can increase due to the offshore bathymetry and this can lead to breaking waves near the shoreline (Li and Raichlen, 2003). Those can run up at the shoreline and destroying coastal regions. It is crucial to understand breaking waves due to tsunamis. Therefore, in order to proceed to the design of sea defense structures and estimate the possible damage resulting from sea submersion due to a tsunami (Li and Raichlen, 2003). This phenomenon, Solitary wave breaking, has been studied in a series of works based on numerous analytical, numerical and experimental studies.

The deployment of the fundamental hydrodynamic equations such as the Navier–Stokes (N–S) equations or Reynolds averaged N–S (RANS) equations is the most appropriate way to investigate the breaking waves (Shao and Changming, 2006). The RANS models have been widely employed and validated in the costal hydrodynamics using the finite difference, finite volume or finite element schemes that are combined with the free surface tracking techniques such as the MAC and VOF methods (Lo and Shao, 2002).

Lemos (1992) simulated a breaking solitary waves and periodic wave breaking on the sloping bed by solving the N–S equations based on SOLC-VOF code coupled with the standard $k-\varepsilon$ turbulence model. Takikawa et al. (1997) investigated a plunging breaker over a sloped bed using both the experimental and numerical analyses. Lin and Liu (1998) presented spilling and plunging breakers by using an advanced RANS modeling. However, in both methods, the Navier – Stokes equations are solved on a fixed Eulerian grid. Problems of numerical diffusion arise due to advection terms in the N – S equations. The diffusion becomes severe when the deformation of the free surface is very large, during which the treatment procedures of the surface cells for capturing the sharpness of the surface becomes complicated (Lo and Shao, 2002).

Particle methods which are among the mesh-free or gridless methods have been widely deployed in many engineering applications as well as the simulation of flow hydrodynamics. Such techniques represent the state of a system as a set of discrete particles, without a fixed Connectivity, followed in a Lagrangian manner. Therefore, particle methods are intrinsically appropriate for the analysis of moving interfaces and free surfaces. Furthermore, fully Lagrangian treatment of

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particles, resolves the problem associated with grid-based calculations by computing the convection terms without the numerical diffusion (Khayyer et al., 2008).

One of the earliest particle methods, the Smoothed Particle Hydrodynamics (SPH) method was first utilized for astrophysical applications (Lucy, 1977; Gingold and Monaghan, 1977). However, it has been extended to model a wide range of engineering applications. The method has also been extended and utilized to simulate the incompressible flows by considering the flow as slightly or weakly compressible with a proper equation of state. Extensive researches have been conducted, based on the SPH method, to display the feasibility of the approach when dealing with the wave and coastal structures.

Monaghan and Kos (1999) simulated run-up and return of a solitary wave traveling over swallowed water and then onto a dry beach backed by a vertical wall by WCSPH with Artificial Viscosity. Lo and Shao (2002) employed ISPH-LES model for the solitary wave impacts against a vertical wall and an inclined slope. Rogers and Dalrymple (2004) employed SPH-LES model for the solitary wave breaking on a beach. Shao and Gotoh (2005) used ISPH model to the simulation of the solitary wave breaking on a beach. Shao (2006), used two-equation $k-\varepsilon$ turbulence model coupled with the incompressible SPH method to examine the spilling and plunging cnoidal wave breaking over a slope. Shao and Changming (2006) devised a 2D SPH–LES model applicable to a cnoidal wave breaking and plunging over a mild slope. Khayyer et al. (2008) proposed Corrected ISPH (CISPH) method and its application to the breaking and post-breaking of solitary waves on a plane slope. Kim and Ko (2008) presented solitary wave propagation on a vertical wall and a sloping wall by WCSPH with Artificial Viscosity. Issa and Violeau (2009) simulated the Plunging Breaking Solitary Wave by WCSPH with various turbulent models, such as constant eddy-viscosity, mixing length and k- model. Ghadimi et al. (2012) generated the Solitary Wave at different wave height to water depth ratios and simulated the breaking of the solitary wave by SPH with Artificial Viscosity.

The turbulence modeling has been of major concern in the study of wave breaking. This paper is intended to apply the 2-D SPS turbulence model of Gotoh et al. (2001) to analyze the wave breaking process on a plane slope. For the current study, a weakly compressible version of the smoothed particle hydrodynamics (WCSPH) method along with a LES approach was used to simulate the wave breaking on a plane slope. In order to investigate the Effects of turbulence models on numerical simulations of wave breaking, the results of present study were compared with experimental results and numerical data found in the literatures. Moreover, to improve the WCSPH results, the Moving Least Squares (MLS) density filter is implemented in the current study. Furthermore, capability of the WCSPH method along with a LES approach to simulate the Solitary wave breaking on a plane slope was compared with result of ISPH method in the literatures.

The SPH method

The SPH method is based on integral interpolants, and we will only refer here to the representation of the constitutive equations in the SPH notation. The main point is to approximately generate any function A(r) with:

$$A(r)=\int_{\Omega} A(r')\delta(r-r')dr' \tag{1}$$

Where $\delta(r-r')$ is the Dirac delta function, r the position vector and r' the sub integral variable. The integral estimate of the exact integral representation of A can be defined by replacing the Dirac delta function with a suitable definition of an interpolation kernel as:

$$A(r)\approx\int_{\Omega} A(r')W(r-r',h)dr' \tag{2}$$

Where r is the vector position; W is the weighting function or kernel; h is called smoothing length.

Using this particle approximation, the following function can be written in discrete notation due to this estimation:

$$A(r)=\sum_b m_b \frac{A_b}{\rho_b} W_{ab} \tag{3}$$

The mass and density are noted by m_b and ρ_b, respectively and $W_{ab}=W(\vec{r}_a-\vec{r}_b,h)$ is the weight function or kernel.
We can get the derivatives of this interpolation by ordinary differentiation

$$\nabla A(r)=\sum_b m_b \frac{A_b}{\rho_b} W_{ab} \tag{4}$$

In the SPH, by using an analytical kernel function, the motion of each particle is computed through interactions with the neighboring particles.

SPH particles move in a Lagrangian coordinates and the advection in N–S equations is directly calculated by the particle motion without the numerical diffusion. Each particle can carry a mass m, velocity u and other properties would vary upon condition (Monaghan, 1992; Monaghan, 1994).

The selection of weighting functions affects the performance of SPH model. They must satisfy some conditions like positivity, compact support. Also:

$$\int_V W(r-r',h)dr'=1 \tag{5}$$

$$\lim_{h\to 0} W(r-r',h)=\delta(r-r') \tag{6}$$

The kernel definition is not unique, and it mainly depends on the knowledge of the investigators (Monaghan, 1992; Liu and Liu, 2010). In this study, the cubic spline function is used, which is generally used and proposed by Monaghan and Lattanzio (Monaghan and Lattanzio, 1985).

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$$W(R,h)=\alpha_d \times \begin{cases} \frac{2}{3}-R^2+\frac{1}{2}R^3 & 0 \le R \prec 1 \\ \frac{1}{6}(2-R)^3 & 1 \le R \prec 2 \\ 0 & R \ge 2 \end{cases} \quad (7)$$

Where $R=\frac{r}{h}$, r being the distance between particles a and b and α_d (the dimensional factor) is $10/7\pi h^2$ in 2D and $1/\pi h^3$ in 3D.

The Lagrangian form of the momentum conservation equation is:

$$\frac{D\vec{u}}{Dt}=-\frac{1}{\rho}\nabla P+g+\upsilon_0\nabla^2\vec{u}+\frac{1}{\rho}\nabla.\vec{\tau} \quad (8)$$

in which, ρ is density, t is time, $\vec{u}$ is velocity, P is pressure, g is gravitational acceleration, υ_0 is kinematic viscosity of laminar flow and τ is Reynolds stress. The pressure gradient term in symmetrical form is expressed in SPH notation as:

$$-\frac{1}{\rho}\nabla P=-\sum_b m_b\left(\frac{P_a}{\rho_a^2}+\frac{P_b}{\rho_b^2}\right)\nabla_a W_{ab} \quad (9)$$

The laminar stress term simplifies to (Lo and Shao, 2002):

$$\left(\upsilon_0\nabla^2\vec{u}\right)=\sum_b m_b\left(\frac{4\upsilon_0\vec{r}_{ab}\nabla_a W_{ab}}{(\rho_a+\rho_b)|\vec{r}_{ab}|^2}\right)\vec{u}_{ab} \quad (10)$$

Where $\vec{r}_{ab}=\vec{r}_a-\vec{r}_b$, $\vec{u}_{ab}=\vec{u}_a-\vec{u}_b$; being $\vec{r}_k$ and $\vec{u}_k$ the position and the velocity corresponding to particle k (a or b) and υ_0 is the kinetic viscosity of laminar flow ($1\times10^{-6}\,m^2/s$).

SPS is deployed to model the effects of turbulence in Sub-Particle Scales (Dalrymple and Rogers, 2006). The eddy viscosity assumption is often used for modeling the SPS stress tensor using Favre-averaging (for a compressible fluid):

$$\frac{\tau_{ij}}{\rho}=2\upsilon_t\tilde{S}_{ij}-\frac{2}{3}\tilde{S}_{kk}\delta_{ij}-\frac{2}{3}C_I\Delta^2\delta_{ij}\left|\tilde{S}_{ij}\right|^2 \quad (11)$$

In which, τ_{ij} is the sub-particle stress tensor, $\upsilon_t=(C_s.\Delta l)^2.|\bar{S}|$ is the turbulence eddy viscosity, C_S is the Smagorinsky constant (0.12), $C_I = 0.0066$, Δl is the particle-particle spacing $|\bar{S}|=\left(2\bar{S}_{ij}\bar{S}_{ij}\right)^{0.5}$ and $\tilde{S}_{ij}$ the element of SPS strain tensor. Therefore, the momentum conservation equation can be written in SPH notation as follow:

$$\frac{D\vec{u}_a}{Dt}=-\sum_b m_b\left(\frac{P_a}{\rho_a^2}+\frac{P_b}{\rho_b^2}+\frac{\tau_a}{\rho_a^2}+\frac{\tau_b}{\rho_b^2}\right)\nabla_a W_{ab} + \sum_b m_b\left(\frac{4\upsilon_0\vec{r}_{ab}.\nabla_a W_{ab}}{(\rho_a+\rho_b)|\vec{r}_{ab}|^2}\right)\vec{u}_{ab}+g \quad (10)$$

The fluid in a standard SPH formulation is assumed to be compressible, allowing the use of an equation of state to determine fluid pressure, instead of solving another differential equation. Fluid density change, in preference to use a weighted summation of mass terms, is calculated as below:

$$\frac{d\rho_a}{dt}=\sum_b m_b u_{ab}\nabla_a W_{ab} \quad (11)$$

As mentioned above, the fluid is considered as a weakly compressible fluid in the standard SPH formulation, facilitating the use of an equation of state for determining the fluid pressure, which is much faster than solving a differential equation like the Poisson's equation. The following equation shows the relationship between pressure and density by Tait's equation of state (Monaghan, 1994):

$$P=B\left[\left(\frac{\rho}{\rho_0}\right)^\gamma-1\right] \quad (12)$$

In which γ is 7, B is $c_0^2\rho_0/\gamma$, ρ_0 is 1000 kg/m^3 the reference density, and c_0 is $c(\rho_0)$,the speed of sound at the reference density.

The pressure field of the particles shows large pressure oscillations, although the dynamics from SPH predictions are generally realistic. Several approaches have been made to overcome this problem. One of the simplest methods is to perform a filter over the density of the particles and the re-assign a density to each particle (Colagrossi and Landrini, 2003). The Moving Least Squares (MLS) approach was used for the current modeling approach.

In this research, the Predictor-Corrector algorithm described by Monaghan (1989) was used in numerical modeling with a time step $\Delta t=1\times10^{-4}$ s. This time step is small enough to satisfy the Courant condition and controlling the stability of force and viscous terms (Monaghan, 1992).

Boundary and Initial conditions

In the SPH model, identification and tracking of free surfaces can always be simply conducted by particles. In the computational domain no special treatment was applied on free surface particles. In fact, the main advantage of the method is that free surface is modeled naturally using the SPH method. For this modeling, the boundary is described by a set of discrete boundary particles. As described in Gomez-Gesteira and Dalrymple (2004), fixed solid boundaries such as the sea bottom and a plane slope were built with two parallel layers of fixed boundary particles set in a staggered manner. In this method the boundary particles share some of the properties of fluid particles, but their velocities are zero and their positions remain constant. For a complete description of the mechanism refer to Crespo et al. (2007). The upstream open boundary is the incident wave boundary. It is modeled by a numerical wave maker composed of wall particles.

In this research the initial velocity of the fluid particles was considered zero and fluid particles were initially placed on a Cartesian grid with dx=dz=0.005 m. An initial density of ρ are assigned for particles based on hydrostatic pressure when the pressure is calculated from the equation of state. So, initial density of a particle (located at depth z) must be calculated taking in account the water column height as follow:

$$\rho = \rho_0 \left(1 + \frac{\rho_0 g (H - z)}{B} \right)^{\frac{1}{\gamma}} \tag{13}$$

which, H is the depth of the tank and z is the distance between the particle and the bottom (Gomez-Gesteira et al., 2005). The initial conditions from the experimental model are set for initial conditions.

The computational system consists of a wave maker at one end of the tank, a sloping plane at the other one. The computational tank was 7.425 (m) long and 0.4 (m) high (Figure 1). Using this initial configuration, the total number of particles in the numerical experiment was 27000, with the particles spaces being set to be 0.005m in two directions.

Wave Paddle

The profile of a solitary wave as a function of distance x and time t is defined as:

$$\eta(x,t) = H_0 \sec h^2 [n(x - Ct)] \tag{14}$$

in which C is the celerity of the wave and n is given by:

$$n = \sqrt{\frac{3H_0}{4h_0^2 (h_0 + H_0)}} \tag{15}$$

$$C = \sqrt{g(H_0 + h_0)} \tag{16}$$

in which, h_0 is the offshore water-depth and H_0 is the deep-water wave height.

Generation of solitary wave by a piston wave maker was performed. The time-dependent wave board trajectory $X(t)$ for producing a solitary wave profile is used the following equation

$$X(t) = \frac{2H_0}{h_0 \beta} \frac{h_0 \tanh(\beta Ct/2)}{h_0 + H_0 [1 - \tanh^2(\beta Ct/2)]} \tag{17}$$

$$\beta = 2\sqrt{\frac{3H_0}{4h_0^2 (H_0 + h_0)}} \tag{18}$$

in which β is decay coefficient (Guizien and Barthelem, 2002).

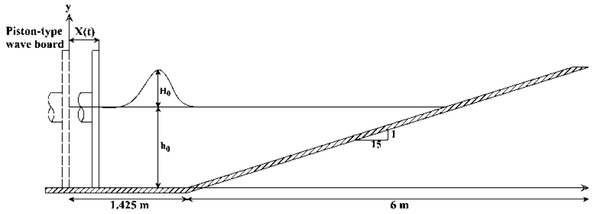

Figure 1. Computational domain for simulation of solitary wave breaking**.**

Figure 2 compares between the analytical and the simulated wave profile for $h_0 = 0.2m$ and $H_0 = 0.07m$. It can be seen that the numerical wave profile agrees well with the analytical one.

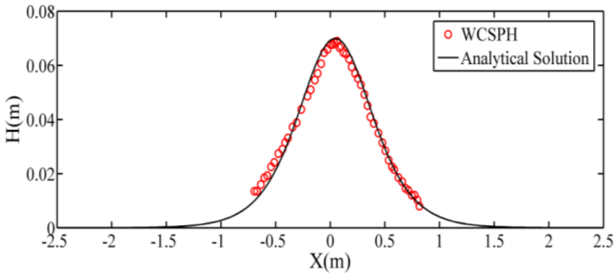

Figure 2. Comparison between the simulated and analytical wave profile for $h_0 = 0.2m$ and $H_0 = 0.07m$.

RESULTS AND DISCUSSIONS

When wave propagates on the slope, it is influenced by shoaling as the depth of water decreases. Hence, the wave profile becomes unsymmetrical, the transmitted wave height increase, the wave crest becomes steeper and eventually it breaks. The turbulence modeling has been of major concern in the study of wave breaking. Then in this step, the Effects of different turbulence models to illustrate the plunging breaking and the splash-up process of a Solitary wave was investigated.

The comparison between experimental and SPH results first shown that SPH was able to simulate breaking plunging wave successfully. Moreover, in Fig. 3, we have compared the results obtained from the present study (LES model) and those reported by Issa and Violeau (2009). They have used a constant eddy-viscosity, a mixing length model and a k – equation model to simulate the solitary wave breaking. Fig. 3 shows that there is no large discrepancy between the four models and it seems that the turbulence effect is not important in the initial steps of the plunging wave process.

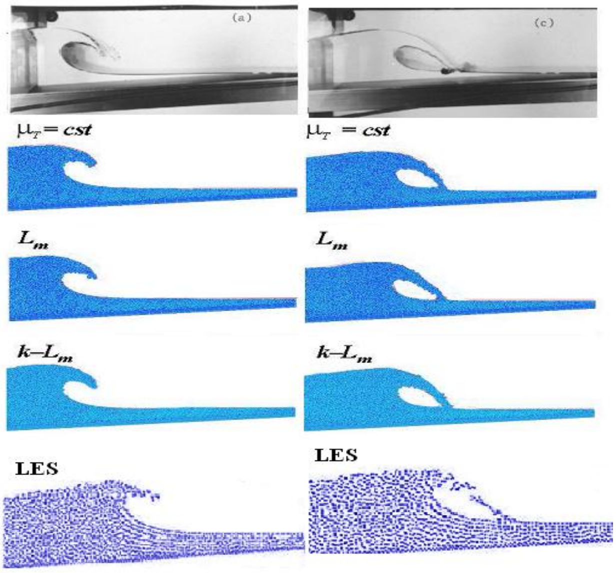

Figure 3. Qualitative comparison of laboratory photographs (Li and Raichlen, 2003) with simulated one achieved by SPH with various turbulence models (Issa and Violeau, 2009) and LES model (present study) at the initial steps of the plunging wave process.

Experimental snapshots of the plunging jet impact on the forward face of the wave have been shown in Fig. 4. The jet generated at the impact point has been directed on shore-ward. This jet impact starts the splash up/run up process. In this case, the jet is reflected at an angle that is more inclined than the corresponding angle of the incident jet (Li and Raichlen, 2003).

SPH simulations of the initial step of the splash up process (SPH snapshots in Fig. 3) show that the model with a LES turbulence model cannot correctly reproduce this phenomenon, while the mixing length model shows slightly better results. Also, the results obtained from the k-equation model are in good agreement with the experiment.

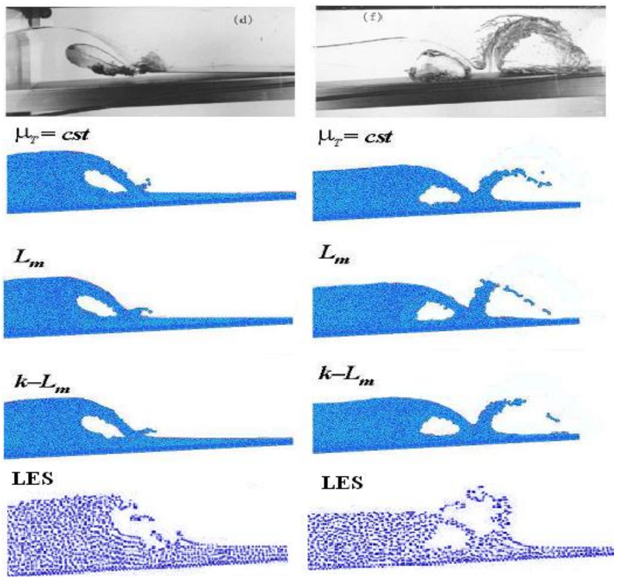

Figure 4. Qualitative comparison of laboratory photographs (Li and Raichlen, 2003) with simulated one achieved by SPH with various turbulence models (Issa and Violeau, 2009) and LES model (present study) at the initial step of the splash up process.

As mentioned above, it can be established that turbulence modeling is important to simulate such phenomenon, because the high shear stress generated in the vicinity of the impinging point leads to high turbulent kinetic energy production rate. Splash up process showed on Fig. 4 reveals that at the presented snapshots all of the turbulence models give fairly good results.

The shape of the reflected jet changes as the incident wave moves shoreward. Also, it curves back toward the incident wave and at the final steps the reflected jet collapses. A comparison between experimental and numerical results in Fig. 5 illustrate that a constant eddy viscosity model is not accurate enough to reproduce the reflected wave.

The other models are slightly better but they have to be improved, especially at the regarding wall treatment. Also, the results obtained from the LES model are in good agreement with the experiment. It is shown that the LES model performs slightly better than other turbulence models to simulate reflected jet.

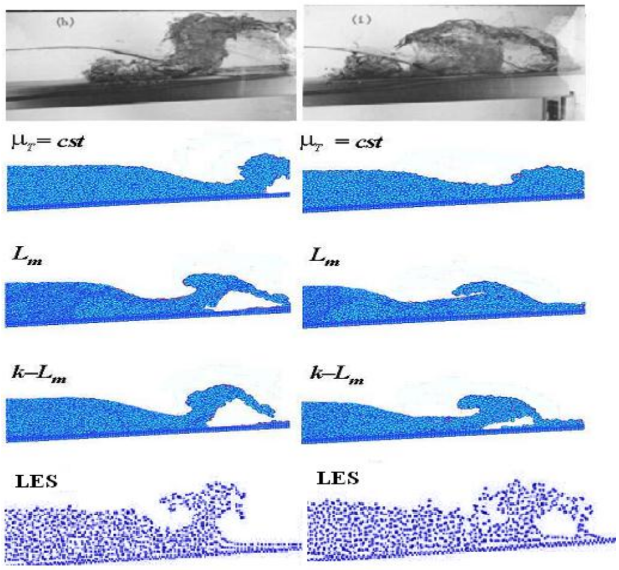

Figure 5. Qualitative comparison of laboratory photographs (Li and Raichlen, 2003) with simulated one achieved by SPH with various turbulence models (Issa and Violeau, 2009) and LES model (present study) at the final stages of splash-up

The space between particles is one of the important factors in the accuracy of the results obtained from SPH method. At this study, we have used a coarser space between particles (0.005 m) than Issa and Violeau (2009), (0.0025 m) that this can affect the final results of the modeling. But we can see that the results obtained from the present study not only have an acceptable accuracy and are very close to those obtained from Issa and Violeau (2009), but at some stages also are slightly better. This shows that the LES model maybe is better choice for modeling the turbulence phenomenon.

The plunging breaking and the splash-up process of a solitary wave illustrates in Fig. 6. In the middle part of the figure, the still photographs are those taken during laboratory experiments (Li and Raichlen, 2003), while, the WCSPH (present study) and ISPH results presented by Khayyer et al., (2008) are shown on the right and left hand sides, respectively.

The WCSPH snapshots show qualitatively well agreement to the laboratory photographs. In general, the model was able to simulate the development and impact of the plunging jet with the resulting splash-up process successfully. On the other hand, the ISPH method could only moderately simulate the development of the plunging jet, while the highly non-linear splash-up process could not be simulated at all.

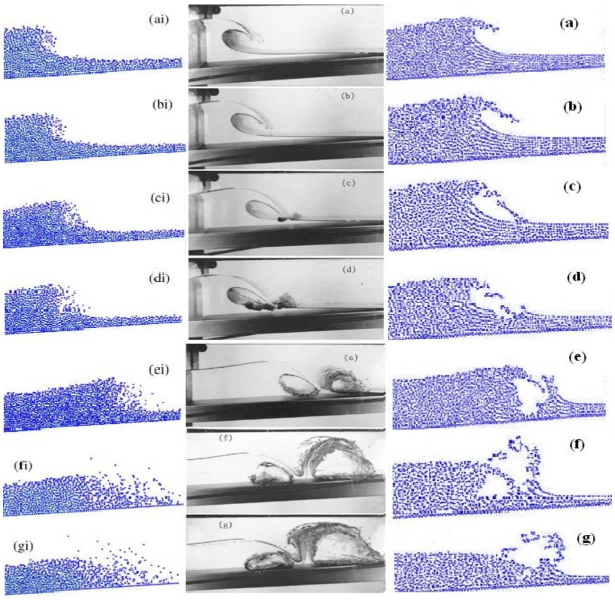

Figure 6. Qualitative comparison of laboratory photographs (center) with ISPH (right) (Khayyer et al., 2008) and WCSPH (left) snapshots.

Simulating this type of flow with a two-phase simulation involving air should increase the quality of the presented results. Nevertheless, when the WCSPH is used to model the details of the highly nonlinear physical processes, implementation of such kind of improvements should be considered.

CONCLUSION

In this study a weakly compressible version of the smoothed particle hydrodynamics (WCSPH) method together with a large eddy simulation (LES) approach was used to simulate the solitary wave breaking on a plane slope. The Effects of different turbulence models to illustrate the plunging breaking and the splash-up process of a Solitary wave was investigated and results showed that the turbulence effect is not important until the splash-up generation, while splash up modeling requires accurate modeling of turbulent effects.

Also, it is shown that the LES model maybe is better choice for modeling the turbulence phenomenon. Moreover, capability of the WCSPH method along with a LES approach to simulate the Solitary wave breaking on a plane slope was compared with result of ISPH method in the literatures. The results shown that the WCSPH coupled with LES model performs better than ISPH method.

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REFERENCES

1. Colagrossi, A. and Landrini, M. 2003. Numerical simulation of interfacial flows by Smoothed Particle Hydrodynamics. Journal of Computational Physics, 191:448–475.
2. Crespo, A.J.C., Gomez-Gesteira, M. and Dalrymple, R.A. 2007. 3D SPH Simulation of large waves mitigation with a dike. Journal of Hydraulic Research, 5: 631–642.
3. Dalrymple, R.A. and Rogers, B.D. 2006. Numerical modeling of water waves with the SPH method. Coastal Engineering, 53:141 – 147.
4. Gingold, R.A. and Monaghan, J.J. 1977. Smoothed particle hydrodynamics: theory and application to non-spherical stars. Monthly Notices of the Royal Astronomical Society, 181: 375–389.
5. Gomez-Gesteira, M., Cerqueiro, D., Crespo, C. and Dalrymple, R.A. 2005. Green water overtopping analyzed with a SPH model. Ocean Engineering, 32: 223–238.
6. Gomez-Gesteira, M. and Dalrymple, R. A. 2004. Using a 3D SPH Method for Wave Impact on a Tall Structure. Journal Waterway, Port, Coastal and Ocean Engineering, 130: 63- 69.
7. Gotoh, H., Shibahara, T., and Sakai, T. 2001. Sub-particle-scale turbulence model for the MPS method Lagrangian flow model for hydraulic engineering. Computational Fluid Dynamics Journal, 4:339–347.
8. Guizien, K. and Barthelem, E. 2002. Accuracy of solitary wave generation by a piston wave maker. Journal of hydrodynamic research, Vol. 40, No.3.
9. Issa, R. and Violeau, D. 2009. Modelling a Plunging Breaking Solitary Wave with Eddy-Viscosity Turbulent SPH Models. CMC, vol.8, no.3, pp.151-164.
10. Khayyer, A., Goth, H. and Shao, S.D. 2008. Corrected Incompressible SPH method for accurate water-surface tracking in breaking waves. Coastal Engineering, 55: 236–250.
11. Kim, N.H., and Ko, H.S. 2008. Numerical Simulation on Solitary Wave Propagation and Run-up by SPH Method. KSCE Journal of Civil Engineering, 12: 221-226.
12. Lemos, C. 1992. Wave Breaking, a Numerical Study, Lecture Notes in Engineering. Springer, Berlin.
13. Li, Y., and Raichlen, F. 2003. Energy balance model for breaking solitary wave run up. Journal of Waterway, Port, Coastal, and Ocean Engineering, 129: 47 – 59.
14. Lin, P.Z. and Liu, P.L.F. 1998. A numerical study of breaking waves in the surf zone. Journal of Fluid Mechanics, 359: 239–264.
15. Liu, M.B. and Liu, G.R. 2010. Smoothed Particle Hydrodynamics (SPH): an Overview and Recent Developments. Archives of Computational Methods in Engineering, Vol 17, Issue 1, pp 25-76.
16. Lo, E. and Shao, S. 2002. Simulation of near-shore solitary waves mechanics by an incompressible SPH method. Applied Ocean Research, 24: 275–286.

17. Lucy, L.B. 1977. A numerical approach to the testing of the fission hypothesis. Astronomical Journal, 82: 1013–1024.
18. Monaghan, J. J. 1989. On the Problem of Penetration in Particle Methods. Journal Computational Physics, 82:1–15.
19. Monaghan, J. J. 1992. Smoothed Particle Hydrodynamics. Annual review of astronomy and astrophysics, 30: 543–574.
20. Monaghan, J. J. 1994. Simulating free surface flows with SPH. Journal Computational Physics, 110: 399–406.
21. Monaghan, J. J. and Kos, A. 1999. Solitary waves on a cretan beach. Journal of Waterway, Port, Coastal, and Ocean Engineering, Vol. 125, No. 3.
22. Monaghan, J. J. and Lattanzio, J. C. 1985. A refined method for astrophysical problems. Astronomy and Aastrophysics, 149:135-143.
23. Ghadimi, P., Abtahi, Sh., and Dashtimanesh, A. 2012. Numerical Simulation of Solitary Waves by SPH Method and Parametric Studies on the Effect of Wave Height to Water Depth Ratio. International Journal of Engineering and Technology, 1: 453-465.
24. Rogers, B.D. and Dalrymple, R.A. 2004. SPH modeling of breaking waves. Proc. 29th Intl. Conference on Coastal Engineering, World Scientific Press, pp. 415 – 427.
25. Shao, S.D. 2006. Simulation of breaking wave by SPH method coupled with $k-\varepsilon$ model. Journal of Hydraulic Research, 3: 338–349.
26. Shao, S., and Changming, J.i. 2006. SPH computation of plunging waves using a 2-D sub particle scale (SPS) turbulence model. International Journal for numerical methods in fluids, 51: 913–936.
27. Shao S.D and Gotoh H. 2005. Turbulence particle models for tracking free surfaces. Journal of Hydraulic Research, 43: 276–289.
28. Takikawa, K., Yamada, F. and Matsumoto, K. 1997. Internal characteristics and numerical analysis of plunging breaker on a slope. Coastal Engineering, 31:143–161.