Journal of Civil Engineering and Urbanism

Volume 4, Issue 4: 474-479 (2014) ISSN-2252-0430

Effect of a High Resolution Finite Volume Scheme with Unstructured Voronoi Mesh for Dam Break Simulation

Hamaidreza Jalalpour* and Seydeh Mona Tabandeh

Department of Civil Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

*Corresponding author's E-mail: hamidrezajalalpour@yahoo.com

ORIGINAL ARTICLE
Received 24 May. 2014
Accepted 24 Aug. 2014

ABSTRACT: In this paper, a high resolution finite volume method (FVM) is developed in order to discretization of multidimensional mathematical equations of Dam Break Phenomena using an unstructured Voronoi mesh grid for evaluating the effect of this type of mesh. Accordingly, the robust local Lax-Friedrichs (LLxF) scheme was used for the calculating of the numerical flux at cells interfaces. The model named FVDBC was run under the asymmetry partial and circular dam break conditions and then verified by comparing the model outputs with the documented results presented in the literature. A statistical analysis using SPSS statistical are performed to confirm the verification of the developed method. In addition, statistical observations indicated a good conformity between the FVDBC and analytical results clearly. The results of FVDBC code with voronoi mesh presented higher efficiency and precision than other meshes. Thus, it is better to use the voronoi mesh in numerical discrete equations because not only able to complicated geometries but also can give accurate results in lower computational effort. Due to a precise agreement between those output and documented results, the FVDBC could be considered as a reliable method for dealing with shallow water (SW) and shock problems, especially those having discontinuities.

Keywords: Dam break, Finite volume method, High resolution, Local Lax–Friedrich scheme, Voronoi Mesh.

INTRODUCTION

Although safety criteria are considered through the design, construction and operation of dams, they may break under unpredictable events such as huge floods, war or earthquake. One of the most important applications of dam break analysis is preparing emergency action plan (EAP). An EAP is a formal plan that determines the potential emergency conditions at a dam and prescribes the procedures that need to be followed in order to minimize the property damage and loss of life. These entail numerical and laboratory investigations of dam break flows and their potential damage. The shallow water equations (SWEs) are conventionally used to describe the unsteady open channel flow such as dam break. These equations are named as Saint Venant equations for one-dimensional (1D) problem and also include the continuity and momentum equations for two dimensional (2D) studies. Many researchers studied the dam break problem, such as Toro (2001), Wu et al. (1999), Wang and Shen (1999), Mohapatra et al. (1999), Zoppou and Roberts (2000), Wang et al. (2000), Wang and Liu (2000), Venutelli (20003), Ponce et al. (2003), Zhou et al. (2004), Liang et al. (2004), Quecedo et al. (2005), Begnudelli and Sanders (2006), Loukili and Soula¨ımani (2007), D´ıaz et al. (2008) and Aliparast (2009), especially using computational fluid dynamic (CFD) methods. Recently, Xia et al. developed a 2D morphodynamic model for predicting dam break flows over a mobile bed (Xia et al., 2010). Erpicum et al. presented a 2D finite volume (FV) multiblock flow solver, which was able to deal with the natural topography variation (Erpicum et al., 2010).

Baghlani utilized a combination of the robust and effective flux-difference splitting (FDS) and flux-vector splitting (FVS) methods to simulate dambreak problems based on FVM on a Cartesian grid. The method combined the effectiveness and robustness of the FDS and FVS methods to precisely estimate the numerical flux at each cell interface (Baghlani 2011). Zhang and Wu developed a hydrodynamic and sediment transport model for dam break flows. The 2D SWEs were solved based on the FVM with an unstructured quadtreemesh grid (Zhang and. Wu, 2011). Singh et al. developed a 2D numerical model to solve the SWEs for the simulation of dambreak problems (Singh et al., 2011).

Chang et al. (2011) proposed a meshless numerical model to investigate the shallow water (SW) dam break in 1D open channel. A numerical model was used to solve the SWEs based on smoothed particle hydrodynamics (SPH). The concept of slice water particles was adapted in the SPH-SWE formulation. Shakibaeinia and Jin (2011) developed a new mesh-free particle model based on the weakly compressible MPS (WC-MPS) formulation for modeling the dam break problem over a mobile bed. Sarveram and Shamsai (2012) investigated the dam break problem in converge and diverge rectangular channels in the unsteady stance using Saint Venant equations and a quasi-Lagrangian method. This paper attempts to present a novel development for 2D dam break problems. A high-resolution FVM is employed to solve the SWEs on unstructured Voronoi mesh. The local Lax-Friedrichs (LLxF) scheme is used for the estimation of fluxes at cells and the numerical approximation of hyperbolic conservation laws.

To cite this paper: Jalalpour H. and Tabandeh SM. 2014. Effect of a High Resolution Finite Volume Scheme with Unstructured Voronoi Mesh for Dam Break Simulation. *J. Civil Eng. Urban.*, 4 (4): 474-479.
Journal homepage: http://www.ojceu.ir/main/

MATERIAL AND METHODS

In this article, the studied domain was discretized using unstructured Voronoi meshes with MATLAB software. In the Voronoi mesh, the chosen point has lower distance in the devoted domain rather than other points. If one point has the same distance from several domains, it will be divided between domains. Indeed, these points create Voronoi cell boundaries. Consequently, internal sections of the Voronoi mesh consist of nodes belonging to one domain and boundaries include nodes that belong to several domains.

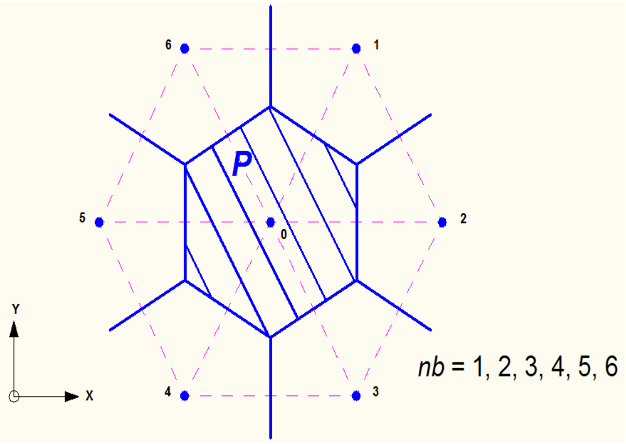

Figure 1. A cell of Voronoi mesh in domain $\boldsymbol{\Omega}$

Governing equation

The continuity and momentum equations of the SW can be written in different forms depending upon the requirements of the numerical solution of governing equations. The 2D SWEs with source terms are given in the vector form considering a rigid bed channel as follows:

$$U_t + F_x + G_y = S \tag{1}$$

$$U_t = \begin{bmatrix} h \\ hu \\ hv \end{bmatrix} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix} \tag{2}$$

$$F_x = \begin{bmatrix} q_x \\ \frac{q_x{}^2}{h} + \frac{1}{2} g h^2 \\ \frac{q_x q_y}{h} \end{bmatrix} \tag{3}$$

$$G_y = \begin{bmatrix} q_y \\ \frac{q_x q_y}{h} \\ \frac{q_y{}^2}{h} + \frac{1}{2} g h^2 \end{bmatrix} \tag{4}$$

$$S = \begin{bmatrix} 0 \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{bmatrix} \tag{5}$$

Where: U is the vector of conserved variables; F and G are the flux vector functions; S is the vector of source terms; u and v are velocity components in the x and y directions, respectively. Also h is water depth and (s_{0x},s_{0y}), are bed slopes in the x and y directions; (s_{fx},s_{fy}) are friction slopes in the x and y directions respectively. The friction slopes are estimated by using the Manning formulas as given in Equation (6):

In the case of the dam break flow, the influence of bottom roughness prevailed over the turbulent shear stress between cells. Therefore the effective stress terms were neglected in the computation.

$$S_{fx} = \frac{n^2 u \sqrt{u^2+v^2}}{h^{1.33}} \quad , \quad S_{fy} = \frac{n^2 v \sqrt{u^2+v^2}}{h^{1.33}} \tag{6}$$

Where

n = Manning's roughness coefficient. In the case of dam-break flow, the influence of bottom roughness prevails over the turbulent shear stress between cells. Therefore the effective stress terms were neglected in the computation (Toro 2001).

Governing equation discretization

In order to solve discrete equations, the parameter in each node was computed considering its discrete equation and newest adjacent nodes' values. Solution procedure can be expressed as follows:

(1) Assuming an initial value in each node as an initial condition.

(2) Calculating the value in a node considering its discrete equation.

(3) Performing pervious step for all nodes over the studied domain, one cycle is performed by repetition this step.

(4) Verifying the convergence clause. If this clause is satisfied, the computing will end otherwise the computations will be repeated from the second step.

In this section, Discretization of the equation 1 is performed applying FVM with unstructured Voronoi mesh in domain $\boldsymbol{\Omega}$. It should be mentioned that F face vertexes nomination direction is counter clockwise from b to a centring o (Figure 2).

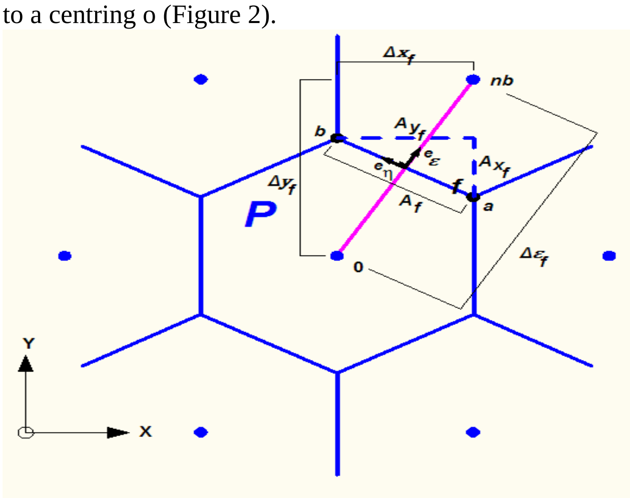

Figure 2. The 2D schematic Voronoi mesh cell used for describing the discretization of the governing equations.

$$\frac{\partial}{\partial t}\int_v u\, d_v + \int_v \vec{\nabla}.H(u)d\vartheta = \int_v S(u)d_v \tag{7}$$

Where

H(u) is input and output flux to voronoi cell that contains G (u) and F (u) functions in x and y directions.

By implementing divergence theorem:

$$\int_v \vec{\nabla}.H(u)d\vartheta = \oint_\Omega n.H(u)d\Omega \tag{8}$$

By applying the Voronoi mesh, (8) is yielded to (9) for an investigated control volume; however, Figure 2 illustrates the 2D Voronoi mesh grid used for describing these equations as follows:

$$\frac{\partial}{\partial t}\int_v u\, d_v + \int_\Omega n.H(u)\, d_\Omega = \int_v S(u)d_v \tag{9}$$

To cite this paper: Jalalpour H. and Tabandeh SM. 2014. Effect of a High Resolution Finite Volume Scheme with Unstructured Voronoi Mesh for Dam Break Simulation. *J. Civil Eng. Urban.*, 4 (4): 474-479.
Journal homepage: http://www.ojceu.ir/main/

$$\frac{\partial u}{\partial t}A + \sum_f \left(\overrightarrow{H(u)} \cdot \vec{A}\right)_f = S(u)\,A \qquad (10)$$

Where,

A is the investigated Voronoi cell area, A_f is the voronoi cells side's area and n is the normal vector. Then,

$$\frac{\partial u}{\partial t} = -\frac{1}{A}\sum_f \left(\overrightarrow{H(u)} \cdot \vec{A}\right)_f + S(u) \qquad (11)$$

$$\vec{A}_f = (A.\vec{e}_\varepsilon)_f \qquad (12)$$

$$\overrightarrow{H(u)} = H(u)_\varepsilon.\vec{e}_\varepsilon + H(u)_\eta.\vec{e}_\eta \qquad (13)$$

Where $\vec{e}_\varepsilon$the unit outward normal vector in each voronoi cell and $\vec{e}_\eta$ is the unit tangent vector in each voronoi cell.

$$\vec{A}_f.\overrightarrow{H(u)} = H(u)_\varepsilon.A_f \qquad (14)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{A}\sum_f H(u)_\varepsilon.A_f + S(u) \qquad (15)$$

$$\int_t^{t+\Delta t}\frac{\partial u}{\partial t}\,d_t = \int_t^{t+\Delta t} -\frac{1}{A}\sum_f H(u)_\varepsilon.A_f\,d_t + \int_t^{t+\Delta t} S(u)\,d_t \qquad (16)$$

$$U_p^{n+1} - U_p^n = -\frac{1}{A}\sum_f H(u)_\varepsilon.A_f\ \Delta t + S(u)\,\Delta \qquad (17)$$

$$H(u)_\varepsilon = n_1.\ F(u)_\varepsilon + n_2.\ G(u)_\varepsilon \qquad (18)$$

$$n_1 = \varepsilon_x = \frac{(x_{nb}-x_p)}{\Delta_\varepsilon} \qquad (19)$$

$$n_2 = \varepsilon_y = \frac{(y_{nb}-y_p)}{\Delta_\varepsilon} \qquad (20)$$

$$\Delta_\varepsilon = \sqrt{(x_{nb}-x_p)^2 + (y_{nb}-y_p)^2} \qquad (21)$$

Where: n_1 is the investigated side cosine in each voronoi cell; n_2 is the investigated side sine in each voronoi cell; x_p, y_p are the evaluated cell center coordinates; x_{nb}, y_{nb} are the neighbor cells centers coordinate; Δ_ε is the distance between the center of investigated voronoi cell and its neighbors.

Discrete equation can be written as equation (22):

$$U_p^{n+1} = U_p^n - \frac{\Delta t}{A}\sum_f [n_1.\ F(u)_\varepsilon.A_f + n_2.\ G(u)_\varepsilon.A_f] + S(u)\,\Delta t \qquad (22)$$

Where:

$F(u)_\varepsilon$, , $G(u)_\varepsilon$ are voronoi cell normal flux vectors and Δt is the interval that computed from courant number clause.

The Local Lax-Friedrichs high order scheme in Voronoi mesh

With expanding equation (22):

$$U_p^{n+1} = U_p^n - \frac{\Delta t}{A} + \sum\left(G_f(v)_{nbout}.A_f.n_{2f}\right) - \sum\left(G_f(v)_{nboin}.A_f.n_{2f}\right)\Big] + S(u)\,\Delta t \qquad (23)$$

We can compute intercell flux by implementing this method as given in Equations (24) to (29):

$$F_f(u)_{nbout} = \frac{f(u_{nb}^n)+f(u_p^n)}{2} - \frac{1}{2}\left|\lambda\left(\frac{u_{nb}^n+u_p^n}{2}\right)\right|\left(u_{nb}^n - u_p^n\right) \qquad (24)$$

$$F_f(u)_{nbin} = \frac{f(u_{nb}^n)+f(u_p^n)}{2} - \frac{1}{2}\left|\lambda\left(\frac{u_{nb}^n+u_p^n}{2}\right)\right|\left(u_p^n - u_{in}^n\right) \qquad (25)$$

$$\lambda = \bar{u} + \sqrt{g\bar{h}} \qquad (26)$$

$$\frac{g(v_{nb}^n)+g(v_p^n)}{2}\ G_f(v)_{nbout} = -\frac{1}{2}\left|\lambda\left(\frac{v_{nb}^n+v_p^n}{2}\right)\right|\left(v_{nb}^n - v_p^n\right) \qquad (27)$$

$$G_f(v)_{nbin} = \frac{g(v_{nb}^n)+g(v_p^n)}{2} - \frac{1}{2}\left|\lambda\left(\frac{v_{nb}^n+v_p^n}{2}\right)\right|\left(v_p^n - v_{nb}^n\right) \qquad (28)$$

$$\lambda = \bar{v} + \sqrt{g\bar{h}} \qquad (29)$$

After computing intercell flux by utilizing Local Lax-Friedrichs scheme, the equation can be solved and the final result can be calculated after each time step. The Δt value should compute by using courant Friedrichs lewy (CFL) after each time step as follow.

$$\Delta t = CFL * min\left(\frac{\Delta\varepsilon}{max|\lambda_1|} + \frac{\Delta\varepsilon}{max|\lambda_2|}\right) \qquad (30)$$

The CFL should range over [0, 1] for achieving to the stability (0 < CFL < 1):

$$\lambda_1 = \bar{u} + \sqrt{g\bar{h}} \qquad (31)$$

$$\lambda_2 = \bar{v} + \sqrt{g\bar{h}} \qquad (32)$$

RESULTS AND DISCUSSION

Validation: The numerical model is validated using the example of dam break for which data is available.

Two dimensional anti-symmetric dam break test: Many researcher, have implemented this test. As shown as Figure 3 the channel that is located in horizontal bed has 200 meters length, 200 meters width it contains an anti-symmetric cut with 75 meters width. The channel and dam domain are assumed frictionless. The upstream water depth is proposed 10 meters and the downstream water depth is assumed 5 meters. In this test we proposed 40*40 node points and the suggested total time for comparing values is 7.2 second. Meshing zone is shown in Figure 4.

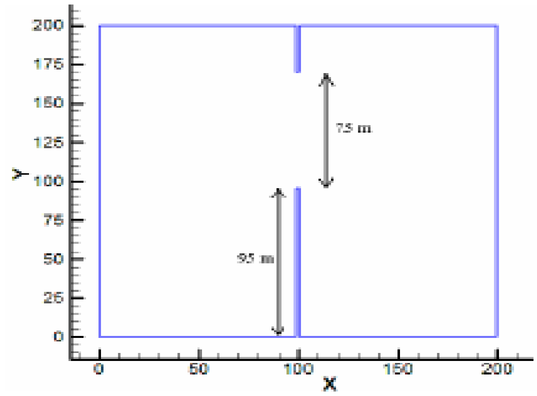

Figure 3. the plan of 2D studied domain for verification

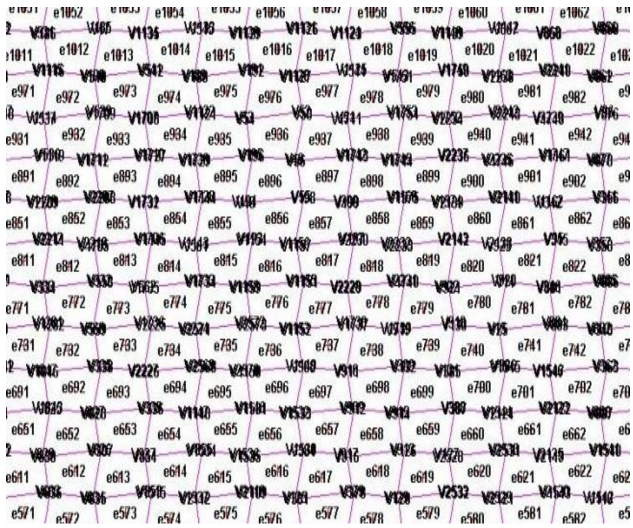

Figure 4. A sample of Voronoi mesh in this model

For mesh configuration comparison, the results of dam break test with Voronoi and rectangular mesh after 7.2 second and references Youssef Loukili and Azzeddine Soula¨ımani (2007) are presented in figure 5, 6 and figure 7 respectively.

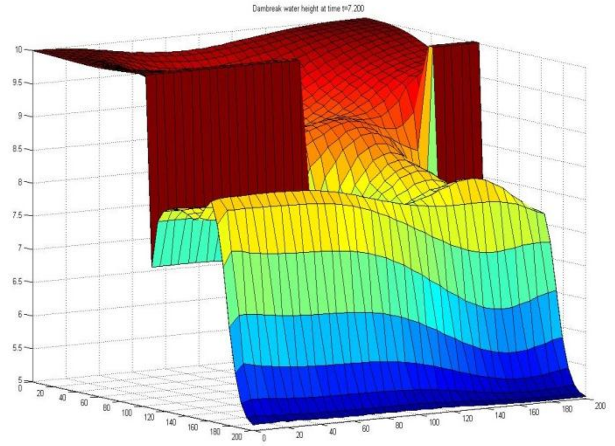

Figure 5. Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second based on presented model (with Voronoi mesh)

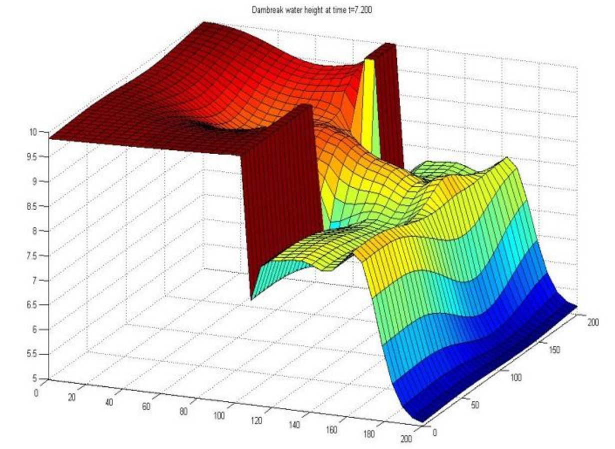

Figure 6. Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second based on presented model (with rectangular mesh)

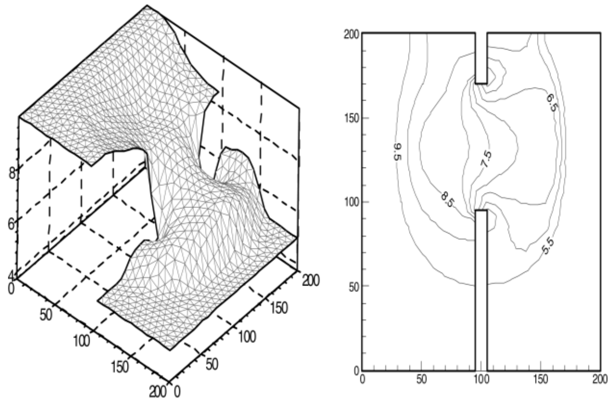

Figure 7. Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second

For a closer look, a diagram of water depth Variation for different type of meshes and references is plotted.

Results of statistical t test analysis to evaluate the effect of various meshes are shown in this table.t test is one of the common and accurate tests that used to compare results. The computed P-Values explain the accuracy of model. If the P-Values closer to 1 more certitude of model achieved.

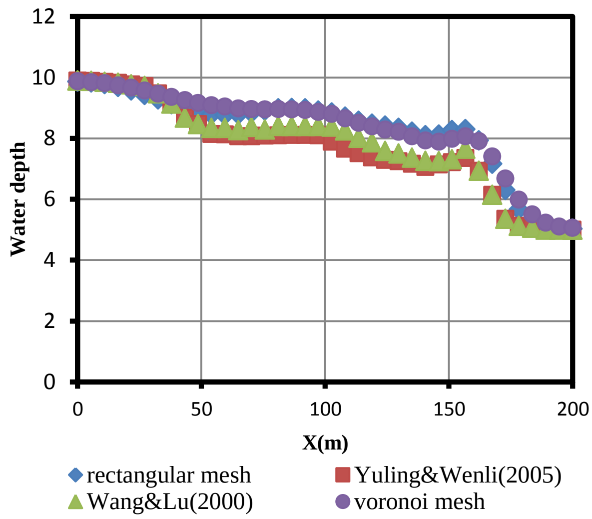

Figure 8. Total results of comparing MATLAB code output with references in two different meshes

Table 1. P-VALUE resulted from statistical t test

Reference	Wang&Lu (2000)		Yuling&Wenli (2005)	
Mesh type	Voronoi	Rectangular	Voronoi	Rectangular
P-Value	0.973	0.967	0.959	0.948

Two dimensional dam break circular test: In this test we have considered a frictionless and horizontal rectangular domain that has 200 meters length and 200 meters width. The initial conditions consisted of two states separated by a circular discontinuity. The computational grid consists of 40 *40 cells and the radius of the circle r = 50 meters and it is centered at x = 100 meters. The water depth outside the circle is 1 meter deep and inside the circle is 10 meters deep. The water depth has shown after 2 second. This result demonstrates that the model is able to simulate complicated geometries.

Figures 9 and 10 illustrate output results of FVDBC code including the water surface formation and water depth contours, respectively.

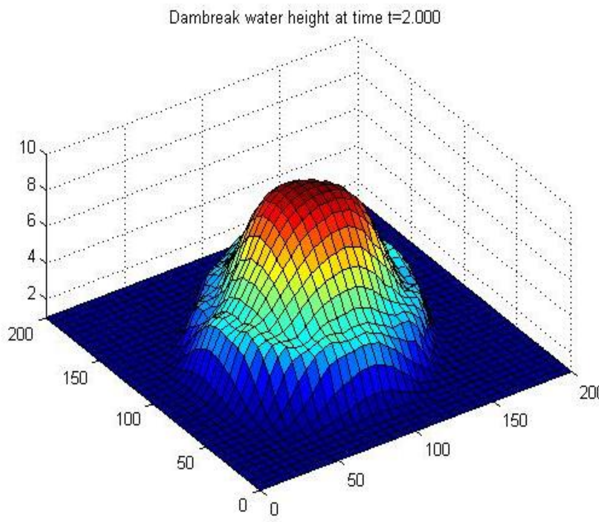

Figure 9. Two-dimensional solution of the circular dam break problem after 2 second based on presented model (with Voronoi mesh)

To cite this paper: Jalalpour H. and Tabandeh SM. 2014. Effect of a High Resolution Finite Volume Scheme with Unstructured Voronoi Mesh for Dam Break Simulation. *J. Civil Eng. Urban.*, 4 (4): 474-479.
Journal homepage: http://www.ojceu.ir/main/

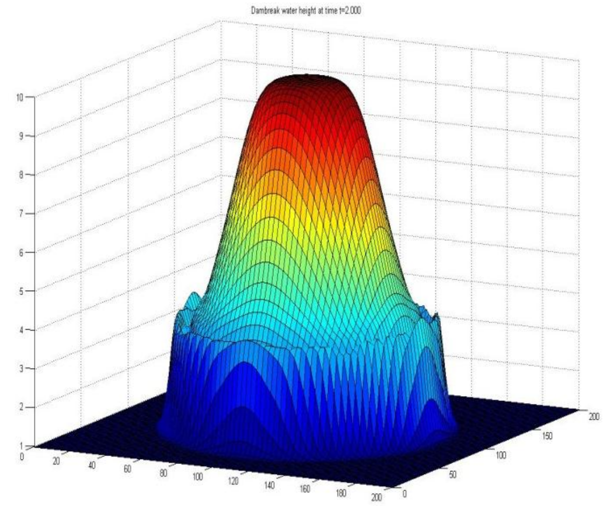

Figure 10. Two-dimensional solution of the circular dam break problem after 2 second based on presented model (with rectangular mesh)

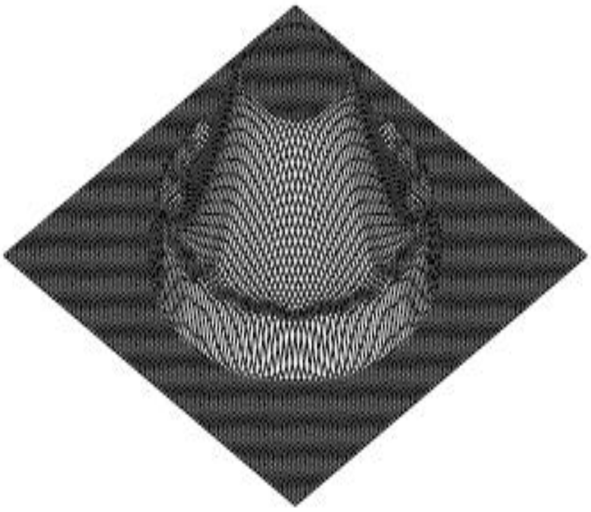

Figure 11. Two-dimensional solution of the circular dam break problem after 2 second based on Christopher Zoppou & Stephen Roberts, 2000.

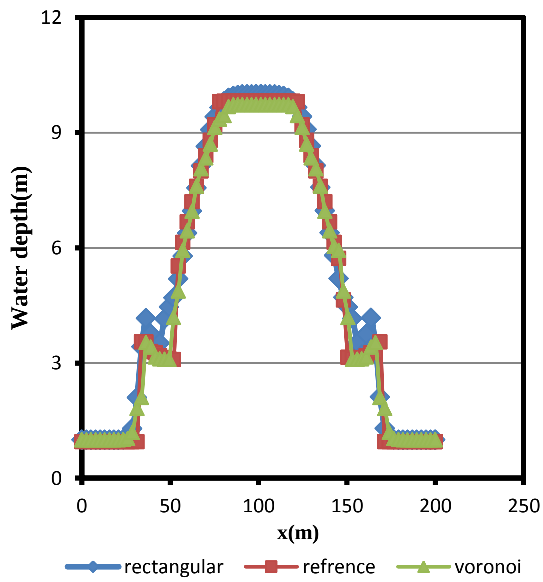

Figure 12. Total results of comparing MATLAB code output with references in two different meshes

For comparing results a diagram of water depth Variation for different type of meshes and references is plotted. We can see that the type of Voronoi mesh is more coincident than the rectangular one. Results of statistical t test analysis to evaluate the effect of various meshes are shown in this table.

Table 2. P-Value resulted from statistical t test

Reference	**Wang& Lu (2000)**	
Mesh type	Voronoi	Rectangular
P-VALUE	0.996	0.974

CONCLUSION

In the present research, a novel and friendly user code was written. It is observed that the LLxF scheme along with the FVM method by using unstructured Voronoi grid is a suitable combination in order to simulate 2D dam break problems. Through the obtained results, there was not, significant numerical dispersion problem or nonphysical alternation. This consequence was predicted because of combination of FVM and Voronoi mesh. The computed P-Values are between 0.95-1 that demonstrates high ability of presented mathematical-numerical model in the evaluation of dam break. FVM have many merits rather than other numerical methods such as, suitable compatibility with studied domain especially in dam break problem, easy prescription for shallow water equation discretization.

REFERENCES

Toro, E.F. (2001). Shock-Capturing Methods for Free Surface Shallow Flows, JohnWiley & Sons, New York, NY, USA

Wu, C, Huang, G, Zheng, Y, (1999). Theoretical solution of dambreak shockwave. Journal of Hydraulic Engineering, 125(11): 1210–1215.

Wang, Z., Shen, H.T. (1999). Lagrangian simulation of one dimensional dam-break flow," Journal of Hydraulic Engineering, 125(11):1217–1220.

Mohapatra, P.K., Eswaran,V, Bhallamudi, S.M. (1999). Two dimensional analysis of dam-break flow in vertical plane, Journal of Hydraulic Engineering, 125(2): 183–192.

Zoppou, C., Roberts, S. (2000). Numerical solution of the two dimensional unsteady dam break, Applied Mathematical Modelling, 24 (7): 457–475.

Wang, J.S, Ni, H.G. and He, Y.S. (2000). Finite-difference TVD scheme for computation of dam-break problems," Journal of Hydraulic Engineering, 126(4): 253–262.

Wang, J.W., Liu, R.X., (2000). A comparative study of finite volume methods on unstructured meshes for simulation of 2D shallow water wave problems, Mathematics and Computers in Simulation, 53(3): 171–184

Venutelli, M. (2003). A fractional-step Pad´e-Galerkin model for dambreak flow simulation," Applied Mathematics and Computation, 134(1): 93–107.

Ponce, V.M., Taher-Shamsi, A, Shetty, A.V. (2003). Dam-breach flood wave propagation using

dimensionless parameters, Journal of Hydraulic Engineering, vol. 129(10): 777–782.
Zhou, J. G, Causon, D.M , Mingham, C.G, and Ingram, D.M. (2004). Numerical prediction of dam-break flows in general geometries with complex bed topography, Journal of Hydraulic Engineering, 130 (4) 332–340.
Liang,Q, Borthwick, A. G. L, Stelling, G. (2004). Simulation of dam- and dyke-break hydrodynamics on dynamically adaptive quadtree grids, International Journal for Numerical Methods in Fluids, 46(2): 127–162.
Quecedo, M, Pastor,M, Herreros, M.I. , Merodo, J.A.F., Zhang, Q. (2005). Comparison of two mathematical models for solving the dam break problem using the FEM method," Computer Methods in Applied Mechanics and Engineering, 194(36-38): 3984–4005.
Begnudelli L, Sanders, B.F. (2006). Unstructured grid finite volume algorithm for shallow-water flow and scalar transport with wetting and drying," Journal of Hydraulic Engineering, vol.132, no. 4, pp. 371–384
Y. Loukili,Y, Soula¨ımani, A. (2007). Numerical tracking of shallow water waves by the unstructured finite volume WAF approximation, "International Journal of Computational Methods in Engineering Science andMechanics, vol. 8, no. 2, pp. 75–88
D´ıaz, M.J.C, Rebollo, T.C, Fern´andez-Nieto, E.D., Vida, J.M, Par´es, C, (2008). Well-balanced finite volume schemes for 2D nonhomogeneous hyperbolic systems. Application to the dam break of Aznalc´ollar, Computer Methods in Applied Mechanics and Engineering, 197(45–48): 3932–3950.
Aliparast, M, (2009). Two-dimensional finite volume method for dam-break flow simulation," International Journal of Sediment Research, vol. 24, no. 1, pp. 99–107
Xia,J, Lin,B, Falconer, R.A, Wang, G. (2010). Modeling dam break flows over mobile beds using a 2D coupled approach, Advances in Water Resources, 33(2) 171–183.
Erpicum,S, Dewals, B.J, Archambeau, P, Pirotton, M, (2010). Dam break flow computation based on an efficient flux vector splitting, *Journal of Computational and Applied Mathematics*, 234(7): 2143–2151.
Baghlani, A. (2011). Simulation of dam-break problem by a robust flux-vector splitting approach in Cartesian grid," Sciatica Iranica, vol. 18, no. 5, pp. 1061–1068
Zhang, M., Wu, W.M. (2011). A two dimensional hydrodynamic and sediment transport model for dam break based on finite volume method with quadtree grid," Applied Ocean Research, 33(4): 297–308.
Singh, J, Altinakar, M.S, Ding, Y, (2011). Two dimensional numerical modeling of dam-break flows over natural terrain using a central explicit scheme, Advances in Water Resources, 34(10): 1366–1375.
Chang, T.J, Kao, H.M., Chang, K.H, SU, M.H.H. (2011). Numerical simulation of shallow-water dam break flows in open channel using smooth particle hydrodynamics, *Journal of Hydrology*, 408(1-2): 78–90.
Shakibaeinia, A., Jin, Y.C, (2011). A mesh-free particle model for simulation of mobile-bed dam break, Advances in Water Resources, 34 (6): 794–807.
Sarveram, H, Shamsai, A, (2012). Modeling the 1D dam-break using Quasi-Lagrangian method, in Proceedings of the 1st International and 3rdNational Conference on Dams and Hydropower, Tehran, Iran.
Loukili, Y., Soula¨ımani, A. (2007). Numerical Tracking of Shallow Water Waves by the Unstructured Finite Volume WAF Approximation, International Journal for Computational Methods in Engineering Science and Mechanics, 8:1–14.
Yuling L., Wenli W. (2005). High Resolution Mathematical Model for simulating 2D Dam Break flow Wave", XXXI IAHR Congress.
Wang J.W., Liu R.X. (2000). A Comparative Study of Finite Volume Methods on Unstructured Meshes for Simulation of 2D Shallow Water Wave problems. Mathematic and computers in simulation.
Zoppou, C., Stephen, R. (2000). Numerical solution of the two dimensional unsteady dam break.