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Presenting Relations for Estimate the Scour Depth Due to Free Falling Jets

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ABSTRACT: High energy dissipation of the free falling jet from the flip-bucket has been always of utmost significance in prevention of the downstream rivers. Scouring due to falling jet incidence with the river bed will threaten the stability of the dam and related structures, while accumulation of eroded sediments can change tailwater elevation and influence the performance of dam or power plants outlets. So determination of the plunge pool dimensions is an important design consideration. The purpose of this study is to develop empirical exponential relations using the principles of statistics and dimensionless relations by applying dimensional analysis in order to estimate the scour depth due to falling jets considering the effective parameters such as discharge, the difference between water levels in the reservoir and tailwater (jet fall height), tailwater depth, the mean diameter of bed particles and flip-bucket jet angle. By comparing the results of the suggested relations in this study with other investigator results it can be indicated that the developed relations have the maximum correlation coefficient and minimum computational error in an appropriate standard deviation range. Also, the accuracy and dispersion of the 139 collected data series that were used for the proposed relations was higher than the previous studies.
Keywords: Dimensional Analysis, Dimensionless Relation, Flip-Bucket, Maximum Scour Depth, Plunge Pool

INTRODUCTION

Scour phenomenon is in fact the displacement of particles from their original location to another one. A particle starts moving when the applied forces by the flow, i.e. the shear and lift forces that separate particles from the bed overcome the weight of the particle. Scour occurs when flow conditions and erodible bed particle and is one of the main causes of damage and failure of hydraulic structures. Despite the long history of scour phenomenon in hydraulics science, since the safe and economic design of hydraulic structures that are located in the flow path requires a proper estimation of the maximum scour depth, presenting a relation that considers all the conditions and existing complexities has been still of particular interest to the Hydraulics and River engineering researchers. Hydraulic structures as flow barriers change the flow pattern nearby, and cause local scour. Investigation of the scour phenomenon becomes important when scour depth is significant so that it reaches the river structure foundation and threatens the stability of the structures or destroys them.

One of the most common methods of dissipating the energy of the flow passing a chute is to use plunge pools and falling jets. Despite the method being economic for the energy dissipation, the falling jet incidence to the downstream river bed will cause scour hole. When the flow is discharged to the pools, the energy is diffused and reduced. Bed scour induced by the jet incidence to the erodible bed with respect to the jet type can be classified into the following groups (Guide No.549 2011).
- Scour due to vertical jets
- Scour due to a horizontal jets

- Scour due to free falling jets
In general, scour phenomenon includes the two following stages:
In the first stage, by displacement of the alluvial materials, jet hydrodynamic forces will break bed materials. In the second stage, separated parts are moved from the original location by the falling water jet and the expansion of the scour hole occurs.
Damages can be prevented and/or minimized by either:
- Prevention of scour;
- Limitation of the scour location and extent.
Due to economic considerations, usually the second option is used, with the goal to control and limit the scoring under the framework of the hydraulic structure. The required estimation of the scour hole dimensions is presented in this paper using one of the three options:
- Prediction of the scouring hole of the natural plunge pool;
- Manufacture of the plunge pools with no concrete cover;
- Manufacture of concrete covered plunge pool (Guide No.549 2011).
Detail design of the plunge pools based on USBR recommendations are given in Figure 1.

Literature review
Numerous empirical relations for prediction of the maximum scour depth downstream of a dam with a jet spillway have been obtained by performing experiments on physical models. Researchers have used different assumptions in developing these relations. Therefore, a wide range of scour depth results are predicted using these

relations. Among the presented relations by the researchers for the scour-hole depth prediction, those capable of predicting a close to reality depth, and also the relations having common parameters with the presented relations in this study are presented in Table 1.

Applied data

The data used in this study (Table 2 and 3) included 95 data sets collected by Azmatullah in order to predict

the scour depth due to falling jet from the flip-bucket chute spillway, and 26 Laboratory data series presented by Kuroiwa-Minaya to investigate scour depth due to falling jet incidence with the non-cohesive bed of the plunge pool. Also, 18 data series collected from physical models of executive dams including Daryan, Azad and Chere dams having Flip-bucket dissipating energy system and falling jet incidence with the plunge pool by Iran Water Research Institute have been used.
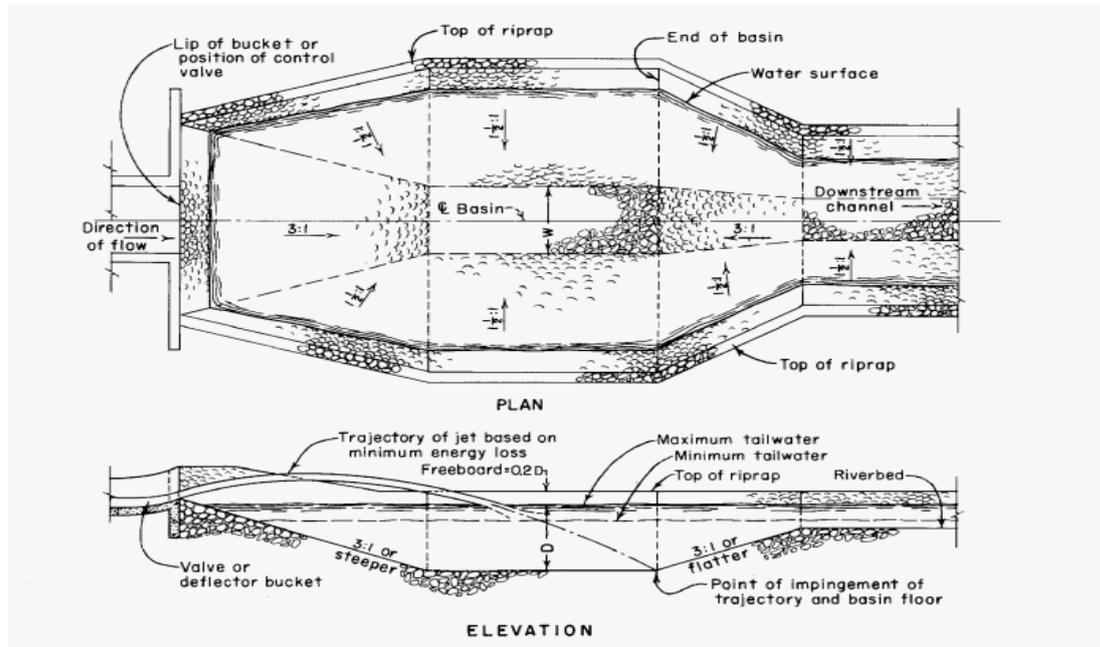

Figure 1. Plunge pool energy dissipater (Design of small dams, 1987)

Table 1. Empirical relations presented by other researchers (Azmathullah et al., 2005; D'Agostino and Ferro, 2004; Ghodsian et al., 2012; Kuroiwa Zevallos and Minaya Espinoza, 2005; Mason and Arumugam, 1985; Veronese, 1937; Yen, 1987)

Researcher	Year	Relations		
Veronese-A	1937	$y_s = 0.202 q^{0.54} H^{0.225} / d_m^{0.42}$	Martins-B	1975	$y_s = 1.5 q^{0.6} H^{0.1}$
Jaeger	1939	$y_s = 0.6 q^{0.5} H^{0.25} \left(\dfrac{y_t}{d_{50}}\right)^{0.333}$	Taraimovich	1978	$y_s = 0.663 q^{0.67} H^{0.25}$
Chee-Padiyar	1969	$y_s = 2.126 q^{0.67} H^{0.18} / d_m^{0.063}$	Sofrelec	1980	$y_s = 2.3 q^{0.6} H^{0.1}$
Chee-Kung	1974	$y_s = 1.663 q^{0.6} H^{0.2} / d_m^{0.1}$	Mason	1985	$y_s = 3.27 \dfrac{q^{0.6} H^{0.05} y_t^{0.15}}{g^{0.3} d_{50}^{0.1}}$
Mason-Arumugam	1985	$y_s = \left(6.42 - \left(3.1 * H^{0.1}\right)\right) \dfrac{q^{(0.6-\frac{H}{300})} H^{0.15+\frac{H}{200}} y_t^{0.15}}{g^{0.3} d_{50}^{0.1}}$		
Yen	1987	$y_s = \left(\dfrac{q^2}{g}\right)^{0.34} \left(6.42 - \left(3.1 H^{0.1}\right)\right) g^{\frac{H}{600}} \left(\dfrac{gH^3}{q^2}\right)^{0.2+\frac{H}{200}} \left(\dfrac{H}{d_{50}}\right)^{0.1} \left(\dfrac{y_t}{H}\right)^{0.15}$		
Azmathullah	2005	$\dfrac{y_s}{y_t} = 6.914 \left(\dfrac{q}{\sqrt{y_t^3 g}}\right)^{0.694} \left(\dfrac{H}{y_t}\right)^{0.0815} \left(\dfrac{R}{y_t}\right)^{-0.233} \left(\dfrac{d_{50}}{y_t}\right)^{0.196} (\phi)^{0.196}$		
Kuroiwa-Minaya	2005	$\dfrac{h_s}{y} = 3.1881 \left(\dfrac{q}{z\left((G-1)g d_{50}\right)^{0.5}}\right)^{0.3875} \left(\dfrac{V}{\sqrt{gy}}\right)^{1.377} \left(\dfrac{z}{H}\right)^{0.6254} \left(\dfrac{d_{85}}{d_{50}}\right)^{0.1185} \left(\dfrac{H}{y_t / sen\theta}\right)^{0.196}$		
Ghodsian	2012	$\dfrac{\varphi}{Y_t} = x_1 Fr_{d90}^{x2} \left(\dfrac{H}{R_H}\right)^{x3} \left(\dfrac{B}{b_1}\right)^{x4} \left(\dfrac{Y_t}{H}\right)^{x5}$	$Fr_{d90} = \dfrac{V}{\sqrt{g d_{90}(\rho_s / \rho - 1)}}$	

Note: y_s: scour depth from tailwater level (*m*), *q:* discharge per unit width (*m²/s*), *g:* acceleration of gravity (*m/s²*), *H:* falling height (*m*), *d₅₀:* average diameter of bed particles (*m*), y_t*:* tailwater depth (*m*), ϕ*:* flip-bucket angle (degree), φ*:* Dimensionless parameter, R_H*:* hydraulic radius, *B:*width (*m*), *Fr :* Froude number, *V:* flow velocity (*m/s*), ρ*:* fluid density (*kg/m³*), ρ_s*:* buoyant sediment density (*kg/m³*).

Table 2. Data collected by researcher (Azamatullah, 2005; Kuroiwa and Minaya, 2005)

Items	Azmatullah	Kuroiwa-Minaya
data	95	26
$q(m^2/s)$	0.0089-0.381	0.033-0.1
$H(m)$	0.2791-1.7962	0.401-0.974
$y_s(m)$	0.0512-0.55	0.11-0.599
$d_{50}(m)$	0.002-0.008	0.0016-0.049
$y_t(m)$	0.0286-0.265	0.05-0.5
$\phi(°)$	10-45	35

Table 3. Data collected from physical models of dams (Final report of Dariyan dam, 2013; Final report of Azad dam, 2008; Final report of Chere dam, 2008)

Items	Dariyan dam	Azad dam	Chere dam
data	6	6	6
$q(m^2/s)$	20.24-143	17-76	20-114
$H(m)$	14.23-142.4	93-96	93.02-98.85
$y_s(m)$	14.2-38.58	19-51	16-46
$d_{50}(m)$	0.0075	0.008	0.008
$y_t(m)$	2.2-15.58	14-23	2.68-7.75
$\phi(°)$	39	46	46

MATERIAL AND METHODS

Presenting exponential and dimensionless relations

The proposed exponential relation using SPSS19 software was modified in order to achieve better results in the form of Equation 1.

Dimensionless relations obtained using empirical models can be generalized to different physical conditions. The purpose of dimensional analysis is to group effective variables of a physical phenomenon into dimensionless groups called Π terms. The advantage of using dimensional analysis is to reduce the number of variables. Among the dimensional analysis methods, Π Buckingham Theorem will be used (Fluid Mechanics-Book of Fox et al., 2004). The proposed dimensionless relation is presented in the form of equation 2. The evaluation results of equations are given in Table 4.

$$y_s = 0.065136 q^{0.563} H^{0.001} d_{50}^{-0.01} y_t^{0.25} \phi^{0.97} \quad (1)$$

$$\frac{ys}{y_t} = 0.0905 \left(\frac{d_{50}}{H} \right)^{-0.001} \left(\frac{q}{\sqrt{y_t^3 g}} \right)^{0.474} \phi^{0.999} \quad (2)$$

In order to compare the data distribution, measured and computed scour depths are presented in Graph 2.

Table 4. Evaluation results of exponential and dimensionless equations

Eq.	R	MAE	RMSE	δ
Eq.1	0.935	0.005	0.109	28.05
Eq.2	0.936	0.005	0.107	28.55

R: Correlation coefficient, MAE: Mean Average Error, RMSE: Root mean square error, δ: average absolute deviation.
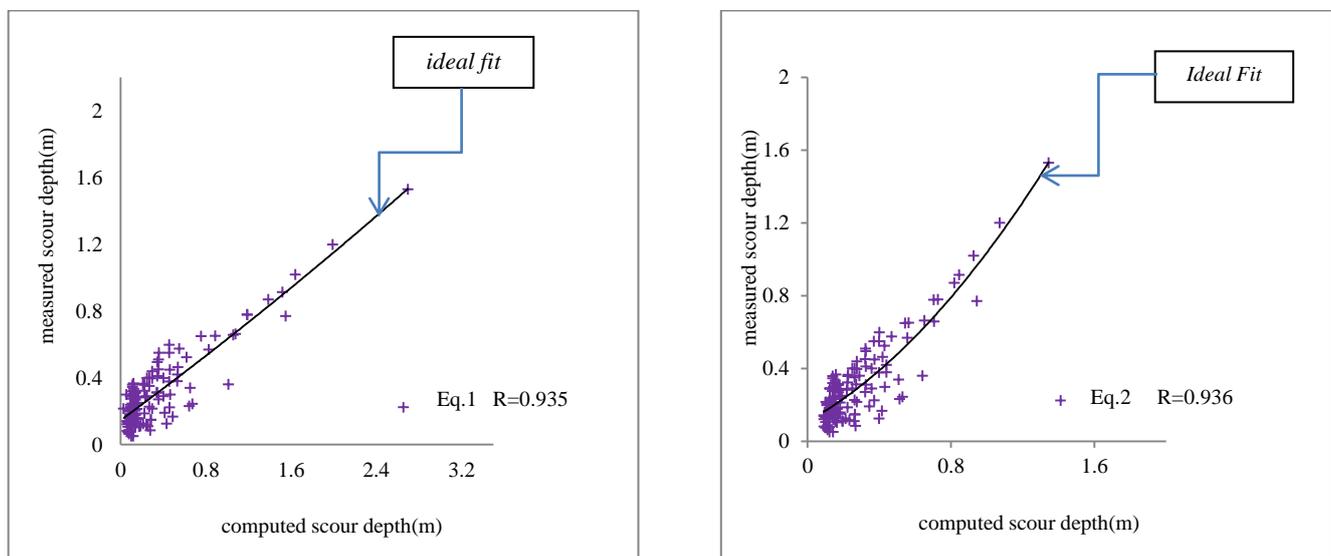
Graph 2. Comparison of the measured and computed scour depths

RESULTS AND DISCUSSION

Evaluation of the presented relations

Comparison and validation of Equations 1 and 2 with the empirical relations having a more suitable data fitting with other presented relations, and also with the recommended relations by the USBR standard and No. 549 Issue is presented in Table 5. Table 5 shows that the Equations 1 and 2 produces the maximum correlation coefficient and the lowest computational error and lowest standard deviation it therefore gives a better scour depth prediction.

Capabilities of the proposed relations

To evaluate the capabilities of the exponential and dimensionless relation proposed in this study, using 139 collected data series, first $r = \dfrac{y_{sm}}{y_s'}$ ratio was calculated, where y_{sm} is the experimental scour depth and y_s' is the measured scour depth by other researchers. The closer the ratio to 1, the more the accuracy of the proposed relation would be. Then, the percentage of the data with r values in the range of 0.5-2 was selected as the model rating (Hoffmans, 1998). Ratings are given in Table 6.

Table 5. Comparison of parameters from Equation 1 with other empirical equations

| Eq.	R	MAE	RMSE	δ
Eq.1	0.935	0.005	0.109	28.05
Eq.2	0.936	0.005	0.107	28.55
Mason	0.85	-49.32	0.19	38.03
Mason-Arumugam	0.852	-51.99	0.2	40.54
chee-Padiyar	0.854	-78.09	0.294	62.97
Martins-B	0.867	-11.63	0.128	28.93
Sofrelec	0.867	-71.17	0.213	50.33
Taraimovich	0.874	-60.99	0.249	63.74
Yen	0.843	-57.35	0.191	40.5

Table 6. Comparison of the proposed equations ratings with the empirical relations ratings

| Evaluated Relations	Ratings (percent)
Azmathullah	96.8
Eq.1	95
Eq.2	93.5
Martins-B	92.8
Mason	81.2
Mason-Arumugam	79.8
Yen	79.1
Sofrelec	77.6
Chee - Padiyar	74.1

Based on the obtained results, the rating of the presented exponential and dimensionless model of the present study was placed after the Azmatullah model rating indicating the high ability of the two models in predicting the scour depth. It is worth mentioning that Azmatullah has obtained his relation based on 95 collected data series and the model rating has been obtained using the same number of data series. Therefore, a high rating for his relation is not unexpected. However, the presented relation by Azmatullah does not define the physical nature of the scour phenomenon properly, because the power of the average grading diameter of the particles parameter is positive and according to this relation, by increasing bed particle diameter scour depth is also increased, which is not a correct definition.

The ratings related to the researchers models are shown in Graph 3. Also, the rating of the proposed exponential and dimensionless model is shown in Graph 4. Based on the graphs and considering the higher density of the points around the 45 degrees line for the proposed exponential and dimensionless relations in comparison to other researchers relations it can be concluded that both models have proper ability to predict the scour depth.
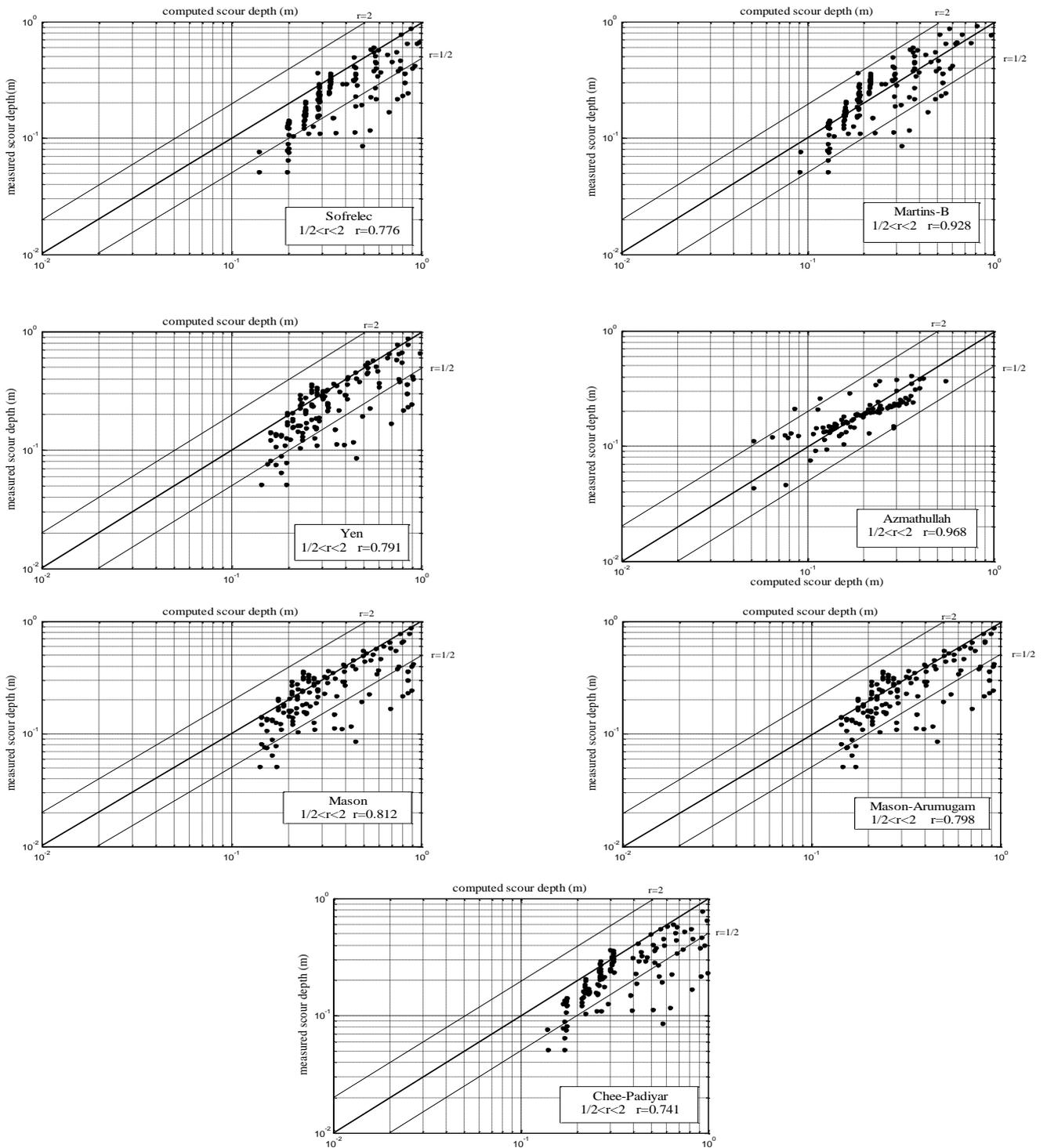
Graph 3. Distribution of the data and ratings of the other investigators relations
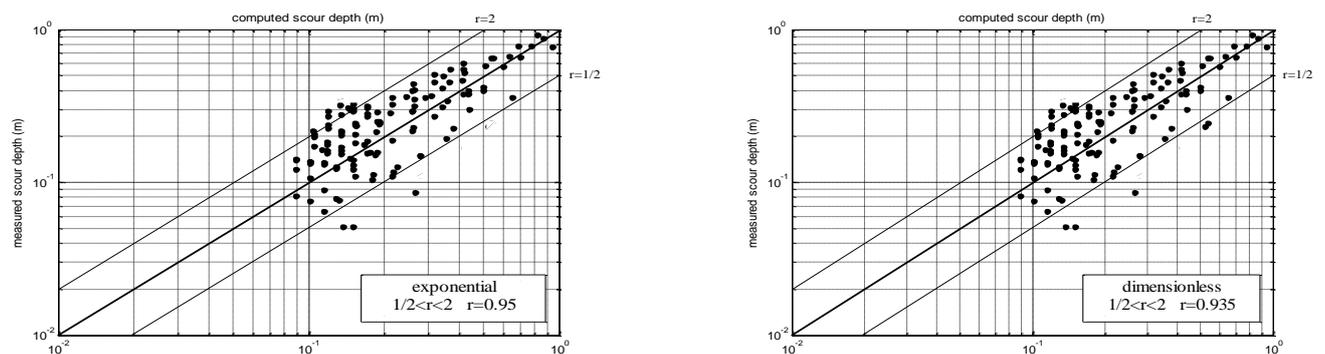
Graph 4. Distribution of data and ratings of the proposed exponential and dimensionless models

Data Analysis

In order to evaluate the effectiveness of each parameter on the proposed exponential and dimensionless relations and to present data distribution pattern on three-dimensional coordinate system, the correlation coefficient of each parameter is given along with the maximum calculated scour depth. The results indicated that the parameter including discharge per unit width is the most effective parameter and has the highest correlation coefficient among other parameters.

Evaluation of the proposed relations

Computational results of the correlation coefficient between scour depth and other parameters are given in Table 7. To show the data distribution on three-dimensional coordinate system, by assuming constant values for calculated scour depth and discharge per unit width, the effect of other parameters is shown in Graph 5.

Scour depth results calculated by the proposed dimensionless relation and data distribution patterns are presented in Table 8 and Graph 6, respectively.

Table 7. Correlation coefficient between calculated scour depth using the proposed exponential relation and other parameters
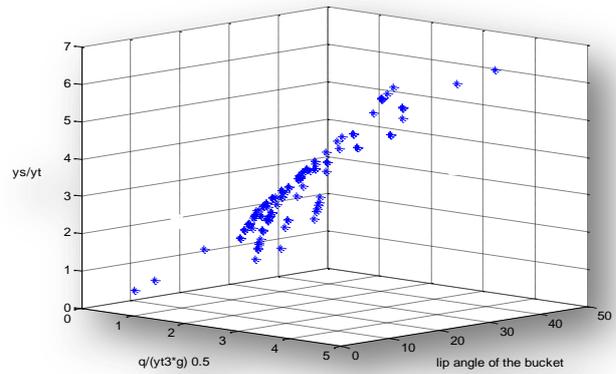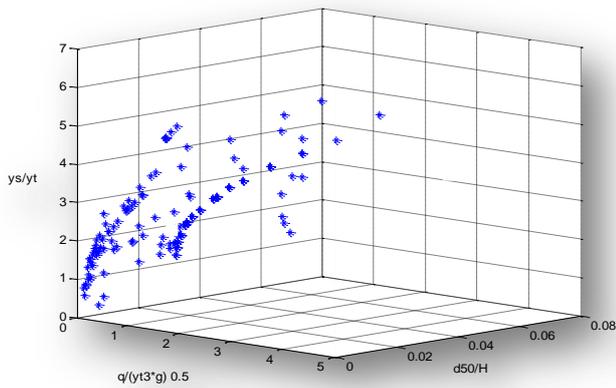
y_s	q	y_t	d_{50}	H	φ
R	0.937	0.827	0.215	0.831	0.741
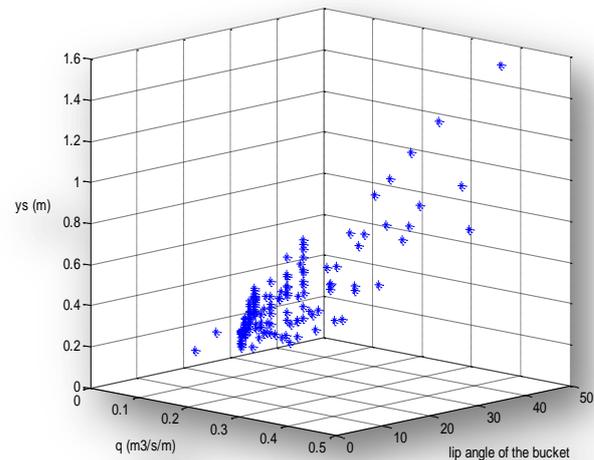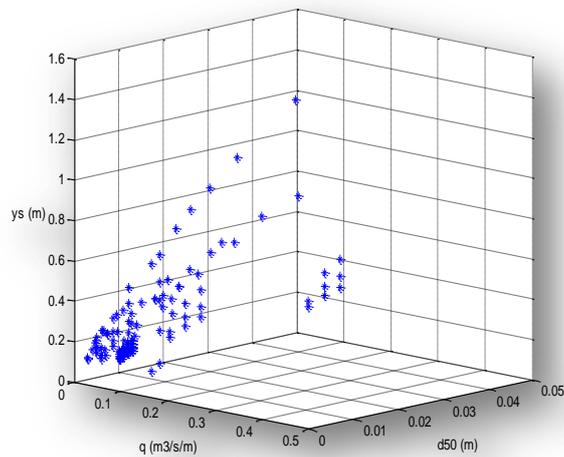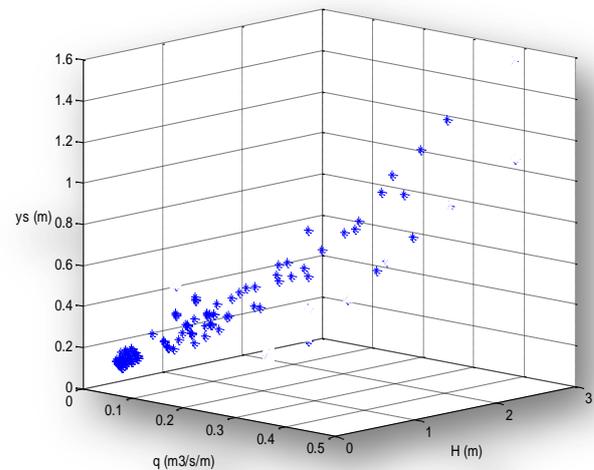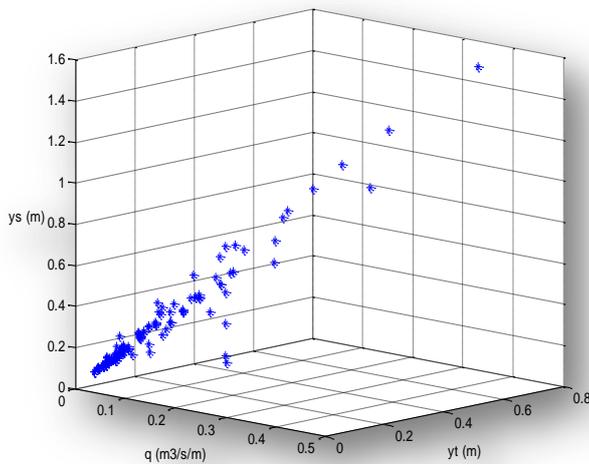
Graph 5. Distribution of calculated scour depths versus other parameters

Table 8. Correlation coefficient between the calculated scour depth by the proposed dimensionless relation and other parameters

y_s	$q/\sqrt{y_t^3 * g}$	d_{50}/H	φ
R	0.911	0.655	0.676

Graph 6. Distribution pattern of scour depth versus other parameters

CONCLUSION

Evaluation of the obtained results by the empirical relations presented by other researchers indicated that the relations could not appropriately predict the scour hole dimensions. Therefore, relations that provide better results are essential. For this reason, exponential and dimensionless relations were presented using 139 collected data series by applying SPSS19 software and Π Buckingham dimensional analysis and modifications to the coefficients and exponents by trial and error. Based on statistical analysis, the proposed relations provide appropriate results. Based on the evaluation of the proposed relations, the followings are concluded:

1- Proposed relations showed a direct relation with the variables such as flow discharge per unit width, falling height, tailwater depth and angle of the flip-bucket so that by increasing each of these variables, scour depth will also increase.

2- The exponent of d_{50} variable is negative for both relations, i.e. by increasing the average bed particle diameter, scour depth is decreased.

3- The variables exponents in both relations are so close so that the results lie in the same numerical range.

4- Interpretation of the confidence range determined shows that a small number of calculated depths are outside the desired range. Therefore, high ratings are assigned to the proposed relations.

5- A higher rating is assigned to the exponential relation in comparison to the dimensionless relation, but data distribution, correlation coefficients, computational error and the standard deviation of the both relations are almost the same.

APPENDIX
Statistical variables to evaluate the relations

To evaluate the accuracy of the proposed relations, the following statistical variables were used:

R	MAE	RMSE	δ
$R = \sqrt{\left(1 - \dfrac{\sum_{i=1}^{N}(o_i - t_i)^2}{\sum_{i=1}^{N}(o_i - \bar{o}_i)^2}\right)}$	$MAE = \dfrac{1}{N}\sum_{i=1}^{N}\lvert o_i - t_i \rvert *100$	$RMSE = \sqrt{\dfrac{\sum_{i=1}^{N}(o_i - t_i)^2}{N}}$	$\delta = \dfrac{\sum \lvert o_i - t_i \rvert}{\sum o_i} *100$

Note: t_i denotes the target values of equilibrium scour depth (m), while o_i and $\bar{o}_i$ denote the observed and averaged observed values of equilibrium scour depth (m), respectively, and N is the number of data points (Azamathullah and Zakaria, 2011).

	Notation
y_s	Scour depth from tailwater level (m)
q	Discharge per unit width (m²/s)
g	Acceleration of gravity (m/s²)
H	Falling height (m)
d_{50}	Average diameter of bed particles (m)
y_t	Tailwater depth (m)
ϕ	Flip-bucket angle (degree)
Π	Dimensionless parameter
r	Proposed model rating
y_{sm}	Experimental scour depth (m)
y_s'	Scour depth measured by researchers (m)
R	Correlation coefficient
MAE	Mean average error
$RMSE$	Root mean square error
δ	Average absolute deviation.

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